

Reynolds Number

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We encountered some interesting dimensionless groups when performing dimensional analysis of fluid mechanical problems. The most prominent dimensionless group that emerges from these analyses is the Reynolds number, named after Osborne Reynolds who made several important contributions to fluid mechanics. One problem Reynolds investigated experimentally is the transition of flow from the orderly kind that we call “laminar flow” to the more chaotic type of flow termed “turbulent flow.” Reynolds published the results of his study in 1883 in an article (1) that is regarded as one of the most important contributions in the field of fluid mechanics. This study changed the way in which knowledge about the transition between the two types of flows, namely, laminar and turbulent, is organized. It was known from earlier studies that turbulent flow occurs in conduits of large cross-sectional dimensions and flows at high velocities, whereas laminar flow occurs in slow flows in conduits of relatively small cross-sectional dimensions. The role of viscosity and density in affecting the type of motion was not as well-characterized. In laminar flow, the pressure drop for flow is linear in the average velocity of the fluid, as we learned earlier, whereas in turbulent flow Reynolds observed that it was approximately proportional to $V^{1.72}$, where V is the average velocity, in his experiments. The actual dependence of the pressure drop on the velocity for turbulent flow in circular tubes is more complicated. For instance the roughness of the interior pipe wall in contact with the fluid profoundly affects the pressure drop, even if it is so minute as not to be visible to the naked eye. We shall learn more about this topic soon in this course.

Through careful experimentation, Reynolds established that the change in the nature of the flow occurs when a certain combination of the parameters in the flow crosses a threshold. Later, this combination was named the “Reynolds number.” For flow in a circular tube of diameter D at an average velocity V , the Reynolds number Re is defined as follows.

$$Re = \frac{DV\rho}{\mu} = \frac{DV}{\nu}$$

Here, μ is the dynamic viscosity of the fluid, and ρ is the density of the fluid. The ratio $\nu = \mu/\rho$ is termed the kinematic viscosity. For circular tubes, the transition from laminar to turbulent flow occurs over a range of Reynolds numbers from approximately 2,300 to 4,000, regardless of the nature of the fluid or the dimensions of the pipe or the average velocity. All that matters is that this specific combination of the parameters, known as the Reynolds number, fall in the range indicated. So, when the Reynolds number is below 2,300, we can expect the flow to be laminar, and when it is above approximately 4,000, the flow will be turbulent. In between these two limits, the flow is termed “transition flow.”

We can ascribe a certain physical significance to the Reynolds number. This is most conveniently seen by recasting this group after multiplying both the numerator and the denominator by the average velocity V .

$$\text{Re} = \frac{\rho V^2}{\mu(V/D)}$$

In this form, we see that the denominator represents a characteristic shear stress in the flow, because it is the product of the viscosity of the fluid and a characteristic velocity gradient obtained by dividing the average velocity by the diameter of the tube. The numerator, in contrast, describes an inertial stress; recall that the larger the density, the more massive a material is, and mass is a measure of inertia. This is the reason for calling the product ρV^2 a characteristic inertial stress. Thus, we can think of the Reynolds number as the ratio of two characteristic stresses in the flow.

$$\text{Re} = \frac{\text{Inertial Stress}}{\text{Viscous Stress}}$$

Stress is force per unit area. Therefore, it is common to express the physical significance of the Reynolds number as follows.

$\text{Re} = \frac{\text{Inertia Force}}{\text{Viscous fore}}$
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We see now that the nature of the flow in a tube, whether laminar or turbulent, depends on the relative importance of the inertia force in comparison with the viscous force. At relatively low values of the Reynolds number, the viscous force is relatively more important, and disturbances in the flow are damped out by viscosity. Thus, it is difficult for disturbances to grow and sustain themselves. On the other hand, at relatively large values of the Reynolds number, the damping of disturbances by viscosity is less effective, and inertia is more important, so that disturbances can perpetuate themselves. This is the basic reason why the Reynolds number serves as a measure for determining whether the flow is laminar or turbulent. Notice that the use of the diameter in the definition of the Reynolds number is an arbitrary choice. It is perfectly legitimate to use the radius of the tube instead of the diameter when performing dimensional analysis, so that the resulting Reynolds number would have been defined using the radius of the tube instead of its diameter, but the value at which transition would occur would have to be altered appropriately. It is for this reason that everyone has agreed on a single convention to avoid confusion. You might wonder why we did not use the length of the tube in defining the Reynolds number. This is because the typical velocity variation in this flow is across the cross-section of the tube, not along its length. Thus, if we had used the length L , the entity $\mu(V/L)$ would not be representative of the shear stress in this flow.

Reynolds number for the steady motion of a sphere through a fluid

When we performed dimensional analysis on the drag experienced by a sphere of diameter d_p moving at a velocity V through a fluid with viscosity μ and density ρ , we encountered a Reynolds number in that problem as well. It is defined as

$$\text{Re} = \frac{d_p V \rho}{\mu}$$

We found that the drag coefficient, which is a dimensionless drag, depends on this Reynolds number. The flow past a sphere is more involved than that in a tube. At high Reynolds number, a boundary layer, in which viscous effects are important, forms on the sphere, and outside of this boundary layer, viscous effects are relatively unimportant, and the flow is dominated by inertia. We shall learn a little more about this topic later in the course when we discuss boundary layers.

Reference

1. O. Reynolds, An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall be Direct or Sinuous, and the Law of Resistance in Parallel Channels, Philosophical Transactions of the Royal Society of London **174**, 935-982 (1883).