An Introduction to Mixing

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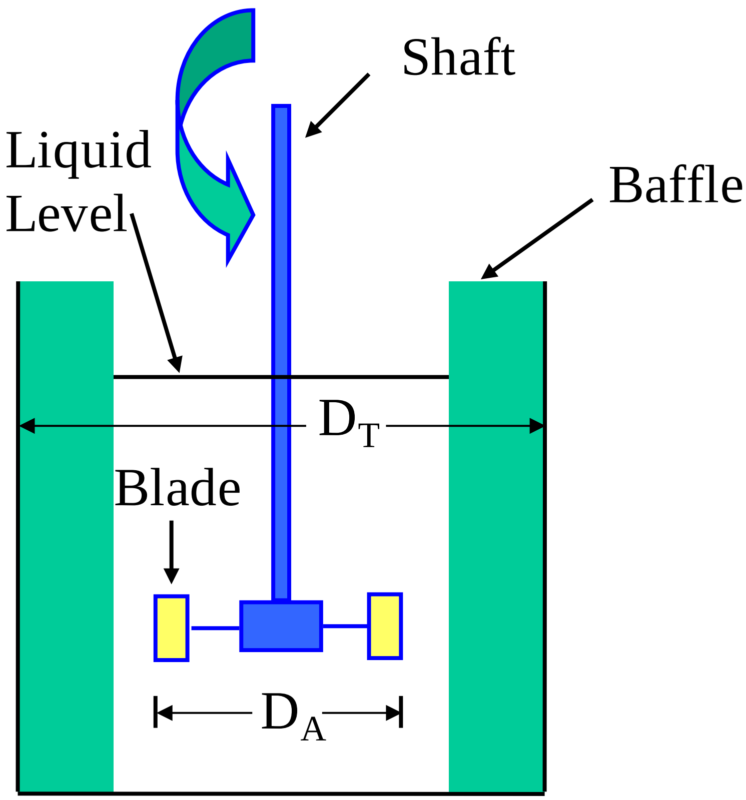
Mixing is a common operation in the chemical process industry. The objectives of mixing are to achieve one or more of the following.

(1) homogenize a mixture of fluids
(2) emulsify one fluid in another
(3) suspend solids in a liquid

A sketch of a generic mixer used for handling low to medium viscosity fluids is given below.



Typically, a mixer is a hollow cylinder filled with the substances to be mixed to some appropriate level. An agitator consisting of a shaft connected to a motor at one end and a set of blades at the other is mounted as shown. In the absence of baffles on the wall, a simple rotating flow, sometimes called a vortex flow, can result that would not cause mixing. In addition, such a simple flow would cause a dip in the surface and this can exacerbate the entrainment of air into the liquid. Therefore, mixers typically have baffles that are attached to the wall to break up this vortex flow. You can learn about mixing and mixers from McCabe et al. (2001) and Holland and Bragg (1995) as well as Perry's

Chemical Engineers' Handbook. Here only a limited discussion is given, focusing on dimensional analysis. One important engineering issue is how to scale up from a laboratory scale device to pilot plant size and on to actual process scale equipment. We use dimensional analysis to find the important groups. A limited list of the important parameters is given below. Several other geometric parameters are omitted. In designing a scale model, it is important to maintain geometric similarity so that all the aspect ratios and shapes are preserved.

ρ: Density of the liquid
μ: viscosity of the liquid
D_T: Diameter of the tank
D_A: Diameter of the agitator
g: acceleration due to gravity
N: Rotation speed
P: Power

By using dimensional analysis, we can show that the Power number N_P will be some function of the Reynolds number Re and the Froude number Fr. It also will depend on the geometric parameters such as the ratio of the diameter of the agitator to that of the tank and those arising from the length scales and shapes of the blades. All of these are assumed to be the same in the model and the prototype and therefore we shall not include them in the list of parameters on which the Power number will depend.

$$N_P = \phi(Re, Fr)$$

Here

$$N_P = \frac{P}{\rho N^3 D_A^5} \qquad Re = \frac{\rho N D_A^2}{\mu} \qquad Fr = \frac{N^2 D_A}{g}$$

Also, the Weber number $We = \frac{\rho N^2 D_A^3}{\sigma}$ can be important in some situations where the objective is to emulsify a mixture; in that case, σ is the interfacial tension between the two liquid phases.

If both the Reynolds and Froude numbers are important, we cannot use a scale model. Let us investigate why. We use the subscripts m and p to designate model and prototype, respectively, in the following. For dynamic similarity,

$Re_m = Re_p$. Therefore,

$$\frac{\rho_m N_m D_{A_m}^2}{\mu_m} = \frac{\rho_p N_p D_{A_p}^2}{\mu_p}$$

Assuming we use the same fluid, $\frac{\rho_m}{\mu_m} = \frac{\rho_p}{\mu_p}$. So, $\frac{N_m}{N_p} = \frac{D_{A_p}^2}{D_{A_m}^2}$.

Now, consider the equality $Fr_m = Fr_p$. This implies

$$\frac{N_m^2 D_{A_m}}{g} = \frac{N_p^2 D_{A_p}}{g} \quad \text{or} \quad \frac{N_m}{N_p} = \sqrt{\frac{D_{A_p}}{D_{A_m}}}$$

So, we must simultaneously satisfy

$\frac{N_m}{N_p} = \frac{D_{A_p}^2}{D_{A_m}^2} = \sqrt{\frac{D_{A_p}}{D_{A_m}}}$. The only way to do this is to set $D_{A_m} = D_{A_p}$. This means that the model must be of the same size as the prototype, which defeats the purpose of building a scale model.

Baffling the system eliminates vortex formation and removes the Froude number as a significant parameter. In that case, dynamic similarity requires that

$$N_m = N_p \frac{D_{A_p}^2}{D_{A_m}^2}$$

assuming we use the same liquid in both the model and the prototype. The implication of this result is that the speed of rotation in the model should be substantially higher than that in the prototype.

Power Requirement for a Mixer

If we restrict attention only to non-vortex forming systems, the data for a given type of mixer can be fitted to

$$N_P = c' Re^a$$

This can be recast as

$$\log N_P = c + a \log Re$$

where the new constant $c = \log c'$. If the flow in the mixer is laminar, it is found that $a = -1$. Therefore, for a laminar mixer, we can write the result for the power input to the mixer as

$$P = c' \mu N^2 D_A^3$$

Laminar mixers are not common. Usually, the flow is turbulent. For a generic baffled mixer, the constant a is found to be small, and when the flow is fully turbulent corresponding to $Re \geq 10{,}000$, we can set $a \approx 0$. In this case, the power input is found to be given by

$$P = 6.3 \rho N^3 D_A^5$$

Note the significantly larger exponents on the rotation speed and the diameter of the agitator in the case of turbulent flow. Another important distinction between the results for the power input in the laminar and turbulent cases is the appearance of the viscosity and not the density in the laminar flow result, and the reverse in the turbulent flow result. Can you rationalize this from your knowledge about the mechanisms of dissipation in these two types of flows?

References

1. F.A. Holland and R. Bragg, Fluid Flow for Chemical Engineers, Chapter 5, Edward Arnold, London, 1995.

2. W.L. McCabe, J.C. Smith, and P. Harriott, Unit Operations of Chemical Engineering, Chapter 9, McGraw-Hill, New York, 2001.