

Introduction to Heat Transfer

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Heat transfer is the study of the flow of heat. In chemical engineering, we have to know how to predict rates of heat transfer in a variety of process situations. For example, in mass transfer operations such as distillation, the overhead vapor has to be condensed to liquid product in a condenser, and the bottoms are boiled off into vapor in a reboiler. Often the feed stream is pre-heated using the bottoms product in a heat exchanger. Another example is the production and use of process steam, which is brought to various locations in a plant through steam pipes as a heating utility. Also, these steam pipes need to be insulated to minimize heat loss to the ambient air. Such insulation is also important when transporting hot fluids from one place to another. A similar application is the transport of refrigerated liquids through piping – here we need to insulate to avoid transferring heat into the liquid from the ambient air. Chemical reactors can generate heat if the reaction is exothermic, and this heat must be removed to avoid a runaway reaction; likewise, endothermic reactions need a supply of heat to maintain the reaction.

Heat transfer also is important in our daily lives. For example, we heat our homes in the winter using hot water in baseboard heaters. We boil water routinely for cooking purposes. If you look inside a modern personal computer, you'll see a fan that is used to cool the electrical circuitry, which becomes warm because of the flow of electrical current through resistances. Sometimes when the circuits are dense, a refrigerant is used in a sealed tube that is boiled at one end where it is warm, to take away the heat, and condensed at the other end where it is cooler.

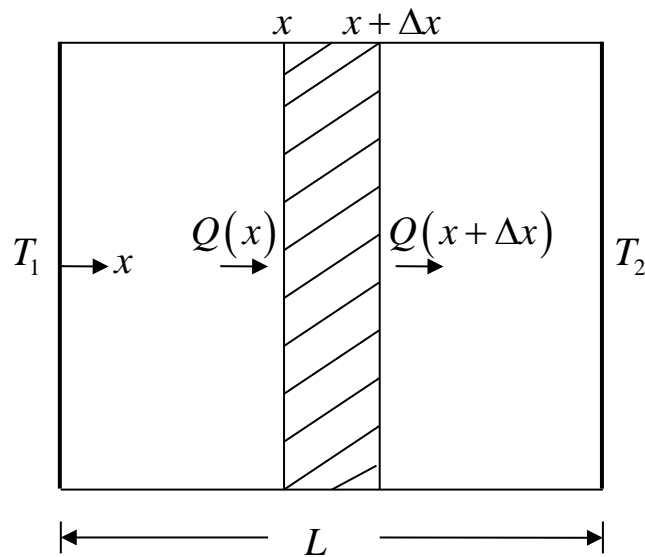
The bodies of warm-blooded animals contain many examples of internal heat transfer as well as heat transfer with the surrounding air. Such animals possess a finely-tuned system that regulates their body temperature by adjusting rates of internal reactions to produce heat, varying blood flow rates as needed, and varying the diameter of blood vessels.

Finally, life on earth is possible only because of energy received from the sun by radiative heat transfer. Radiative heat transfer also is important in the manufacture of steel and other such materials, and in furnaces used for melting glass.

In all of these situations and many others, we can identify three basic mechanisms of heat transfer. They are conduction, convection, and radiation. Next, we discuss each of these mechanisms in some detail. You'll find additional information in your textbook. Also, two good references are the texts by Mills (1) and Holman (2).

Conduction

Conduction is an electronic/atomic mechanism of transferring energy from one place to another in solids, and a molecular mechanism of heat transfer in liquids and gases. We begin with a simple example of heat conduction through a window when the inside of the room is warm and the outside is cold. Let us assume that we know the temperatures of the inside and outside surfaces of the window, and build a steady state model of heat flow by conduction through the window. We model the window as a tall and wide rectangular slab, and refer to it as a slab in subsequent discussion, because the model is applicable to many other situations besides heat transfer through a window. A sketch is given below.



We assume that the temperature is uniform in the other two coordinate directions y and z in the slab, varying only with distance x from the left boundary shown in the sketch. The heat flow rate through the slab is assigned the symbol Q . The heat flow rate can vary with x , a possibility that is acknowledged by writing it as $Q(x)$. Incidentally, the textbook by Welty et al. uses the lower case symbol q for this heat flow rate; most others use capital Q for the heat flow rate, and lower case q to designate the heat flow rate per unit area, or heat flux. We shall follow that convention in the course.

Our initial objective is to determine how $Q(x)$ varies with x at steady state. Eventually, we expect to determine the temperature distribution in the slab and the heat flow rate through it. We first consider two surfaces within the slab that are parallel to the left and right surfaces. These imaginary surfaces are located at some distance x from the left wall, and a slightly larger distance $x + \Delta x$ as shown. At steady state, there can be no accumulation of energy in the volume bounded by these two surfaces shown as a hatched region in the sketch; if energy accumulates in that volume, its temperature would change with time, and this would lead to unsteady conditions, violating the assumption of steady state. Therefore, we conclude that the rate of heat flow into the surface at x must be exactly the same as the rate of heat flow out of the surface at $x + \Delta x$.

$$Q(x) = Q(x + \Delta x)$$

Let us rearrange this result after division by Δx .

$$\frac{Q(x + \Delta x) - Q(x)}{\Delta x} = 0$$

Now take the limit as $\Delta x \rightarrow 0$. This leads to the simple differential equation

$$\frac{dQ}{dx} = 0$$

Integration is straightforward, and leads to the result

$$Q = \text{constant}$$

Of course, this is a common-sense conclusion that we could have drawn merely by physical inspection of the problem. The reason for going through the mathematical development is to learn how to construct such models, which will prove useful in more complicated situations.

The heat flow rate $Q = qA$, where A is the cross-sectional area of the slab normal to the x -direction, and q is the heat flux. Therefore, we reach the conclusion that the heat flux q must remain constant, independent of x . If we wish to determine this heat flux from a knowledge of the temperatures of the two surfaces, we must have a model that relates the heat flux to temperatures. Such a model is called Fourier's law, which is analogous to Newton's law of viscosity, which we encountered in fluid mechanics. Newton's law relates the momentum flux or stress to the gradient in velocity. Likewise, Fourier's law relates the heat flux to the gradient of temperature. Fourier postulated that the heat flux is proportional to the local temperature gradient. In simple one-dimensional form, we can write

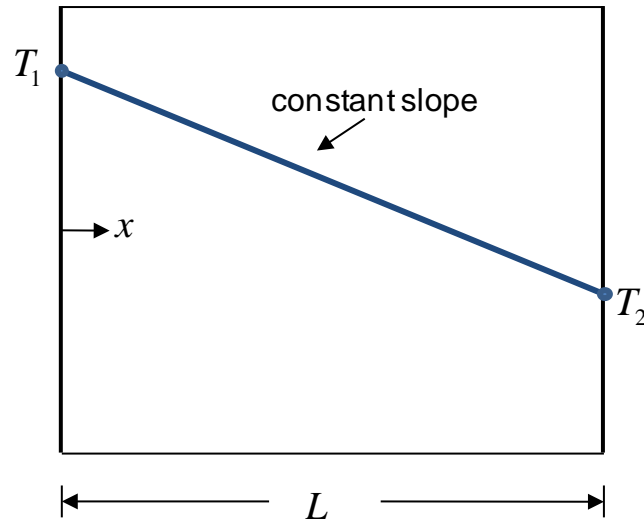
$$q_x = -k \frac{dT}{dx}$$

where we have introduced a subscript x for the heat flux to indicate that it is the component of the heat flux in the x -direction. Often, the subscript x is dropped for convenience, and we simply write

$$q = -k \frac{dT}{dx}$$

Because q is a constant, the right side must be a constant as well. We'll discuss the quantity k , called the thermal conductivity of the material of the slab, which appears in Fourier's law, a bit

later. For now, let us just treat it as a known constant. We can see that dT/dx within the slab must be a constant. This is an important conclusion. It tells us that the temperature profile in the slab must be a straight line.



The temperature gradient is the constant slope of the straight line shown in the figure. It is obtained as

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L - 0} = -\frac{T_1 - T_2}{L} = -\frac{\Delta T}{L}$$

where $\Delta T = T_1 - T_2$ is considered the driving force for conduction through the slab. The heat flux can be written as

$$q = -k \frac{dT}{dx} = k \frac{\Delta T}{L}$$

and the steady heat flow rate through the slab, $Q = qA$, is obtained as

$$Q = kA \frac{\Delta T}{L}$$

This can be recast as

$$Q = \frac{\Delta T}{L/kA} = \frac{\text{Driving Force}}{\text{Resistance}}$$

Fourier's Law

Because there are no temperature variations in the y and z -directions in the problem we just modeled, the components of the heat flux in those two directions are zero, but in general heat flux has a direction in space as well as a magnitude, so that it is a vector. Therefore, the general form of Fourier's law is a vector relationship between the heat flux vector and the temperature gradient vector.

$$\mathbf{q} = -k \nabla T$$

To see how we were able to write the simpler version earlier, we need to write each side in the above vector relationship in terms of its components in the basis set $(\mathbf{i}, \mathbf{j}, \mathbf{k})$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the three coordinate directions, (x, y, z) , respectively. Thus,

$$\mathbf{q} = \mathbf{i} q_x + \mathbf{j} q_y + \mathbf{k} q_z = -k \left[\mathbf{i} \frac{\partial T}{\partial x} + \mathbf{j} \frac{\partial T}{\partial y} + \mathbf{k} \frac{\partial T}{\partial z} \right]$$

By matching the coefficients of \mathbf{i}, \mathbf{j} , and \mathbf{k} between the left and right sides, we can write results for the individual heat flux components in each coordinate direction. This is how we obtained the simpler version we used earlier. You may notice that we used an ordinary derivative in writing Fourier's law in the x -direction.

$$q_x = -k \frac{dT}{dx}$$

This is because the temperature in the slab varies only in the x -direction, so that the partial derivative becomes the ordinary derivative in that context.

You may wonder why there is a negative sign in the right side in Fourier's law, $\mathbf{q} = -k \nabla T$. The gradient of a scalar function points in the direction of maximum rate of increase of that function at a given point in space. But, heat flows downhill – that is, the local heat flux is in the direction of maximum rate of decrease in temperature, exactly opposite to that of the temperature gradient. This is why a minus sign is introduced in the proportionality written above.

$$\boxed{q = -k \frac{dT}{dx}}$$

Thermal Conductivity

The symbol k stands for a material property; it is the thermal conductivity of the medium. We can use Fourier's law to infer the units and dimensions of thermal conductivity.

$$\text{Thermal Conductivity} = \frac{\text{Heat Flux}}{\text{Temperature Gradient}}$$

Therefore k has units of $\frac{W/m^2}{K/m} = \frac{W}{m \cdot K}$ in SI. In the British system, k is measured and reported in $BTU/(hr \cdot ft \cdot ^\circ F)$. You should take the time to work out the dimensions of the thermal conductivity from basic principles as $ML/(T^3\theta)$, where $M, L, T,$ and θ stand for the dimensions of Mass, Length, Time, and Temperature Difference, respectively. Approximate values of the thermal conductivities of a few common materials at $27^\circ C$ from (1) are listed below.

Material	k $W/(m \cdot K)$
copper	386
aluminum	204
stainless steel	15
pyrex glass	1.09
water	0.611
fiberglass	0.038
polystyrene	0.028
air	0.027

We shall abbreviate “thermal conductivity” to “conductivity” because there should be no confusion with electrical conductivity in the present context. The conductivity of solids, with the exception of “insulators” is typically larger than that of liquids, and the conductivity of liquids is usually larger than that of gases. The conductivity of insulators is similar to that of gases.

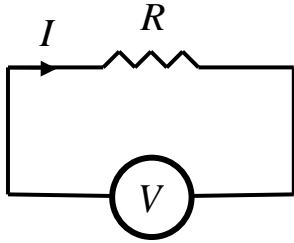
Thermal conductivity depends on temperature. Typically, in gases, the conductivity increases as the temperature is increased, while it decreases in liquids as the temperature is increased. In some liquids, it is nearly constant over a wide range of temperature values, and in a few liquids it increases with temperature over some range before decreasing, thus displaying a maximum. There is no general rule for the temperature dependence of the thermal conductivity of a solid. The conductivity of copper decreases as the temperature is increased, whereas that of aluminum increases with increasing temperature. Good electrical conductors also prove to be good thermal conductors. Some solids such as fiberglass, polystyrene, or wood, which are poor conductors of electricity, also have a very low thermal conductivity, and are considered thermal insulators.

Thermal conductivity is relatively insensitive to pressure variations, except when the material is near critical conditions.

Electrical Analogy and Conduction Through a Composite Slab

Earlier, we wrote the heat flow rate through a slab as the ratio of a driving force to a resistance.

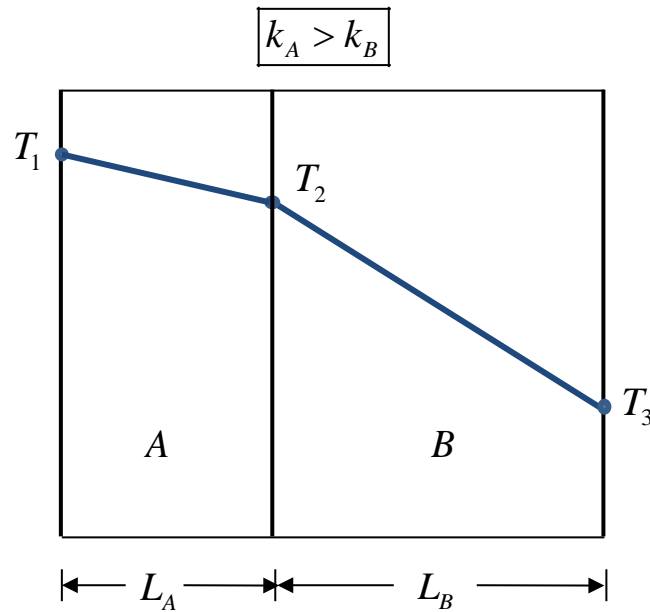
$$Q = \frac{\Delta T}{L/(kA)} = \frac{\text{Driving Force}}{\text{Resistance}} . \text{ One can draw an analogy with a simple electrical circuit.}$$



When a potential difference V is applied across a resistance R , the current through the circuit is

$$I = \frac{V}{R}$$

Now, consider steady conduction through a composite slab, made of two different materials A and B of different thermal conductivities k_A and k_B , respectively.



By performing an analysis similar to that given earlier, we can quickly conclude that at steady state, the heat flow rate through the composite slab must be constant, independent of time and position in the slab. Therefore,

$$Q = Q_A = Q_B$$

We can see that Q_A and Q_B are given by

$$Q_A = k_A A \frac{T_1 - T_2}{L_A} \quad Q_B = k_B A \frac{T_2 - T_3}{L_B}$$

where A is the area for heat flow. Therefore, we can write

$$Q = \frac{T_1 - T_2}{L_A / (k_A A)} = \frac{T_2 - T_3}{L_B / (k_B A)} \quad \text{or} \quad Q = \frac{T_1 - T_2}{R_A} = \frac{T_2 - T_3}{R_B}$$

where $R_A = L_A / (k_A A)$ and $R_B = L_B / (k_B A)$ are the individual resistances of slabs A and B to heat conduction, respectively. Thus, the heat flow rate is given by the driving force within each segment, divided by the resistance of that segment. We can write

$$T_1 - T_2 = Q R_A \quad T_2 - T_3 = Q R_B$$

Adding, we find

$$T_1 - T_3 = Q (R_A + R_B)$$

or

$$Q = \frac{T_1 - T_3}{R_A + R_B}$$

It is evident that the heat flow rate through a composite slab is simply the total driving force divided by the sum of the two resistances. The analogy to a series electrical circuit should be evident.

The linear temperature profiles in each segment of the composite slab are shown in the sketch for the case $k_A > k_B$. For a constant heat flux, when the conductivity is large, the slope of the temperature profile must be small, and vice versa. You should try sketching the temperature profiles in the reverse case $k_A < k_B$. Also, recognizing that the thermal conductivity actually depends on temperature, try making a sketch of the temperature profile for steady conduction in a single slab when the thermal conductivity increases or decreases with increasing temperature (Hint: the profile will no longer be a straight line).

It is straightforward to extend the results to any number of segments in the composite slab. Thus, we can write in general

$$Q = \frac{\Delta T}{\sum_k R_k}$$

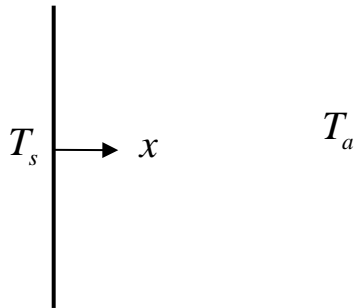
where ΔT is the temperature difference across the entire composite slab, and $R_k = L_k / (k_k A)$ is the resistance of the k 'th segment to conduction.

Convective Heat Transfer

We already are familiar with fluid motion. When an element of fluid moves from one place to another, it brings its energy content with it, so that this is another mechanism for transferring energy from one place to another. Convection can be forced, which means that we are using some means to cause the motion – this can be a pressure difference in a conduit that is generated by a pump or a storage tank at some elevation, or by a fan blowing air, and so on. The alternative to forced convection is free or natural convection, which refers to flow that occurs naturally, without any intervention. Natural convection is caused by the action of density gradients in fluids in conjunction with the gravitational force. Typically, we see less dense fluid rise when it is located next to more dense fluid, while the more dense fluid sinks. Thus, a baseboard heater at home warms the neighboring air, and the warm air rises, allowing cooler air to move toward the heater and get warmed in turn. Incidentally, the “radiator” in older homes, is really not a radiator, but is similar to a baseboard heater that uses natural convection in its operation.

Natural convection is the source of all of the weather patterns that we experience. It also causes mixing in oceans and lakes. Density differences that lead to natural convection can arise from temperature variations in a fluid in a heat transfer context, or from composition variations, which can occur in mass transfer equipment.

Regardless of the reason for the motion in the fluid, whether forced or natural, we call this mechanism “convective heat transfer.” Now, consider a hot surface at a temperature T_s in contact with a moving fluid that is at some other temperature T_a far from the surface.



In this situation, there will be a heat flux from the surface to the fluid. We write this heat flux as

$$q_x|_{x=0} = h(T_s - T_a) = h \Delta T$$

where h is termed the “convective heat transfer coefficient” between the surface and the fluid, and $\Delta T = (T_s - T_a)$ is the driving force for heat transfer. Note that this equation only serves to define this heat transfer coefficient h , and we must have some way of knowing its value to use the equation for the heat flux. Later, we’ll learn about correlations that are used to calculate values of h in a variety of flow situations.

We note that the tangential and normal velocity components are both zero at a solid surface that is stationary and impervious to the fluid. Therefore, heat transfer to the fluid at the surface actually occurs only via conduction, neglecting radiative heat transfer. It is therefore possible to write

$$q_x|_{x=0} = -\left(k_{fluid} \frac{\partial T}{\partial x}\right)_{x=0}$$

Therefore, if we have some means of calculating the temperature gradient in the fluid in contact with the solid surface, we can predict the value of q_x , and therefore, the heat transfer coefficient between the solid and the fluid. In simple flow situations, this is precisely what one does, but making such predictions are beyond the scope of the present course.

The units and dimensions of the heat transfer coefficient can be inferred from its defining equation.

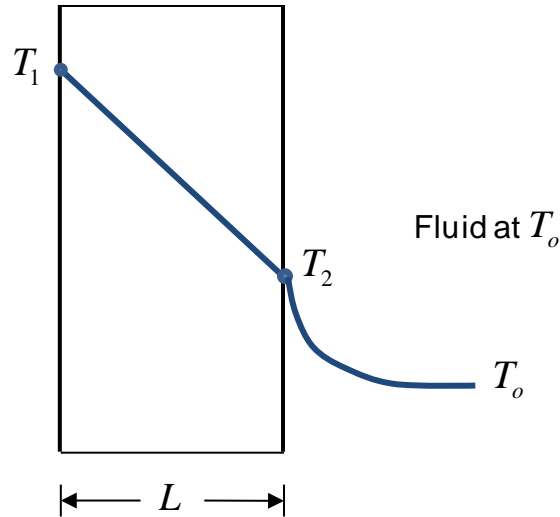
$$\text{Heat Transfer Coefficient} = \frac{\text{Heat Flux}}{\text{Temperature Difference}}$$

Therefore, h has units of $\frac{W/m^2}{K} = \frac{W}{m^2 \cdot K}$ in SI. In the British system, h is measured and reported in $\frac{BTU}{hr \cdot ft^2 \cdot ^\circ F}$. The dimensions of h are $M/(T^3\theta)$, where M, T , and θ stand for the

dimensions of Mass, Time, and Temperature Difference, respectively. Depending on the intensity of the motion in the fluid and the properties of the fluid, the convective heat transfer coefficient can vary in value over at least five orders of magnitude. In a given geometry, the heat transfer coefficient will also depend on position, but we usually use average values over the entire heat transfer surface. Typically, the heat transfer coefficient is relatively small for natural convection in gases caused by small density differences, takes on intermediate values for forced convection in gases and liquids, and is large for phase change applications such as boiling and condensation heat transfer. For example, for forced convection in air, a typical range of values of h might be $20-250 W/(m^2 \cdot K)$. For forced convection in water, the range might be $50-10^4 W/(m^2 \cdot K)$. Values of h for free convection heat transfer are much smaller. The range for air is roughly $3-50 W/(m^2 \cdot K)$, while values of h for free convection in water lie in the approximate range $10-10^3 W/(m^2 \cdot K)$. In boiling and condensation, h varies from $3 \times 10^3 - 10^5 W/(m^2 \cdot K)$.

In forced convection, the heat transfer coefficient is independent of ΔT , the driving force for heat transfer. In natural convection situations, however, h will depend on ΔT , because it is the temperature difference that causes density variations, and leads to the flow. Typically, $h \propto \Delta T^{1/n}$ where n varies from 3 to 4. In phase change heat transfer, such as boiling and condensation, the driving force is chosen as the difference between the actual temperature of the surface and the saturation temperature of the fluid under the prevailing conditions. Therefore, in these situations, h will again depend on ΔT .

We close with the concept of additivity of resistances in steady heat flow situations when conduction and convective heat transfer occur in series. Consider the case where heat is conducted through a slab and is transferred to the surrounding fluid on the outside by convective heat transfer. A sketch of the system is shown below.



The two sides of the slab of width L are at temperatures T_1 and T_2 as shown. The ambient fluid is at temperature T_o , and heat transfer between the right surface of the slab and the ambient fluid is described by a heat transfer coefficient h . At steady state the same amount of heat Q must flow through the slab and out to the fluid. From the models we have constructed, we can write Q as

$$Q = qA = k_{solid}A \frac{T_1 - T_2}{L} = hA(T_2 - T_o)$$

where q is the steady state heat flux and A is the cross-sectional area of the slab for the flow of heat. We can rewrite the above result as

$$Q = \frac{T_1 - T_2}{L/(k_{solid}A)} = \frac{T_2 - T_o}{1/hA}$$

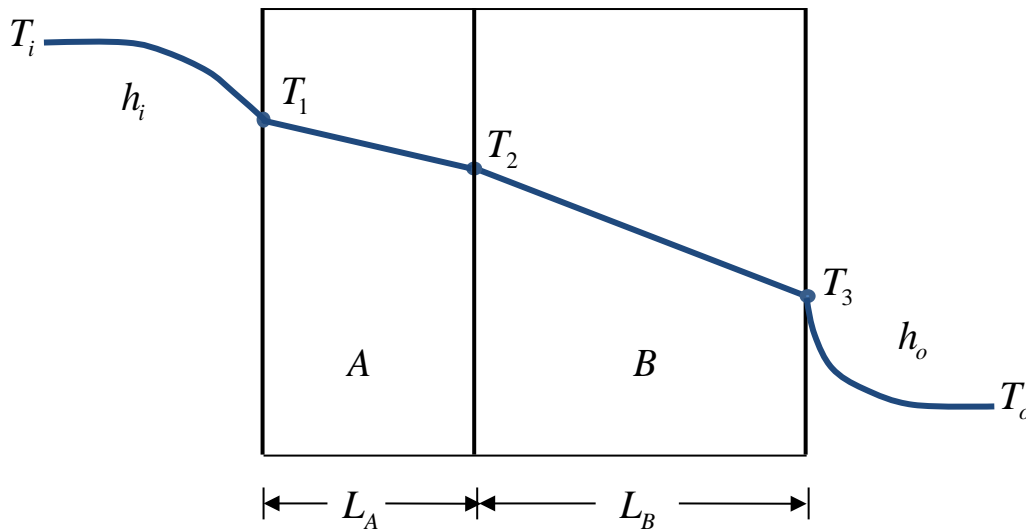
in the form of the ratios of individual temperature difference driving forces and resistances. Here, $R_{solid} = L/(k_{solid}A)$, and $R_{fluid} = 1/(hA)$. By the same techniques used in the case of steady conduction through a composite solid, we can show that

$$Q = \frac{T_1 - T_o}{R_{solid} + R_{fluid}}$$

Thus, the concept of adding resistances in series works equally well for convective heat transfer resistances as it does for conduction resistances.

Multiple Resistances in Series – The Overall Heat Transfer Coefficient U

We can extend the idea developed above for one conduction resistance and one convective heat transfer resistance in series in a straightforward manner to multiple resistances within the slab and convective heat transfer resistances on both sides. Consider a composite slab made of two materials that is exposed to air on the inside of a room on one side and outside air on the other side. The convective heat transfer rate on the inside is described using a heat transfer coefficient h_i , and that outside likewise by h_o . Let the temperature inside the room away from the surface at T_1 be T_i , and that in the ambient air outside be T_o . The various surface temperatures in the slab are marked on the sketch below.



At steady state, we can write

$$Q = h_i A (T_i - T_1) = k_A A \frac{T_1 - T_2}{L_A} = k_B A \frac{T_2 - T_3}{L_B} = h_o A (T_3 - T_o)$$

which can be recast as

$$Q = \frac{T_i - T_1}{1/h_i A} = \frac{T_1 - T_2}{L_A / (k_A A)} = \frac{T_2 - T_3}{L_B / (k_B A)} = \frac{T_3 - T_o}{1/h_o A}$$

or

$$Q = \frac{T_i - T_1}{R_i} = \frac{T_1 - T_2}{R_A} = \frac{T_2 - T_3}{R_B} = \frac{T_3 - T_o}{R_o}$$

where the convective resistance to heat transfer on the left side of the composite slab $R_i = 1/(h_i A)$, and the convective resistance to heat transfer outside is $R_o = 1/(h_o A)$. The two conduction resistances are $R_A = L_A / (k_A A)$ and $R_B = L_B / (k_B A)$. As before, by writing results for each individual temperature difference as the product of the heat flow rate and the resistance of

that portion in the path of heat flow, and adding the temperature differences, it is straightforward to establish that

$$Q = \frac{T_i - T_o}{\sum_k R_k}$$

where the summation over the index k involves adding the various resistances defined here. This result is conveniently recast as

$$Q = \frac{T_i - T_o}{1/(UA)}$$

or simply as

$$\boxed{Q = UA(T_i - T_o)}$$

where the overall heat transfer coefficient U is obtained from

$$\frac{1}{UA} = \sum_k R_k = \frac{1}{h_i A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_o A}$$

This equation states that the overall resistance to heat transfer, signified by $1/(UA)$ is comprised of contributions from each individual resistance to heat transfer in series. Other resistances can be added as needed, for example when there is a thermal contact resistance between the two segments in a composite slab, or when fouling occurs in an industrial heat transfer situation.

There is a completely analogous description that applies to mass transfer problems, where we shall define an overall mass transfer coefficient in a similar manner.

Radiation Heat Transfer

Radiation heat transfer is ubiquitous, because all matter emits and absorbs electromagnetic radiation. The electromagnetic radiation spectrum is huge, but heat transfer is mostly concerned with a small part of it, called thermal radiation. The wavelength of this radiation is in the approximate range of $0.1\text{--}100\ \mu\text{m}$, and includes the visible portion, in the approximate range of $0.35\text{--}0.75\ \mu\text{m}$. Because of the limited time we have in this course, we shall not be able to spend much time on radiative heat transfer. All we can do is to introduce some definitions, and discuss very simple cases of radiative heat transfer between two surfaces.

All matter above absolute zero emits radiation. In a solid, radiation that is emitted by a layer is re-absorbed within the next few atomic layers. Thus, our main concern in describing radiation heat transfer between objects is to consider radiation emitted by a solid surface and radiation absorbed by that surface. We define irradiation G as the radiant energy that is incident on a solid surface per unit area per unit time. Thus, the units of G in SI would be W/m^2 . Likewise, we define the radiosity J of a surface as the radiant energy emitted by a surface per unit area per unit time. The units of J also are W/m^2 .

Now, we define a “black body.” This is a surface that absorbs every bit of radiant energy that is incident on it, regardless of wavelength or incident angle. It does not reflect any of the incident radiant energy. Therefore, all of the radiation that leaves the surface of a black body must be emitted by it. It is given by a simple relationship called the Stefan-Boltzmann Law.

$$J = \sigma T^4$$

In this equation, σ is the Stefan-Boltzmann constant, which has a value of approximately $5.6704 \times 10^{-8}\ W/(m^2 \cdot K^4)$, and T is the absolute temperature of the surface in Kelvin. Interestingly, a nearly black body that absorbs all thermal radiation incident on it need not appear black to the human eye, which is sensitive to electromagnetic radiation only in a very narrow part of the spectrum comprising visible wavelengths.

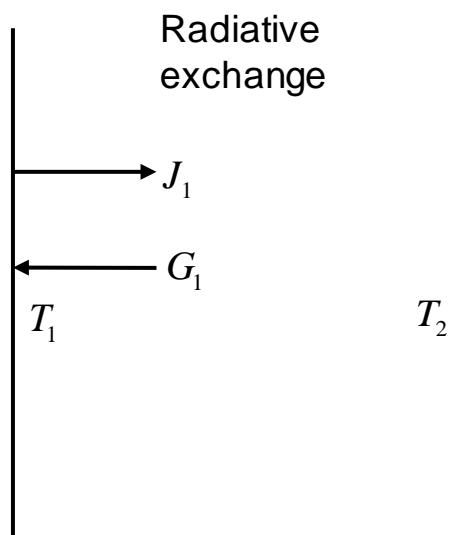
Actual surfaces are not perfectly black. They are called “gray.” Most practical surfaces in the context of heat transfer are opaque – that is they absorb a fraction of the incident radiation and reflect the rest. The fraction absorbed α is called “absorptivity” or “absorbance” and the fraction reflected ρ is called “the reflectivity” or “reflectance.” Both depend on the wavelength, but in a simplistic description, we integrate over all wavelengths of interest. Because all incident radiation is either absorbed or reflected, the absorptivity and reflectivity of a gray surface must add up to unity.

$$\alpha + \rho = 1$$

The radiosity of a gray body is given by $J = \varepsilon \sigma T^4$ where ε is called the “emissivity” or the “emittance” of the surface. For a black body, which is a perfect emitter, $\varepsilon = 1$. It is possible to show that the emissivity of a gray body must be equal to its absorptivity. That is, $\varepsilon = \alpha$.

Radiation heat transfer between two long and wide surfaces facing each other

Consider the extremely simple case of two long and wide flat surfaces at two different absolute temperatures facing each other.



The surface on the left is at an absolute temperature of T_1 while the surface on the right is at T_2 . Let us assume that both are black.

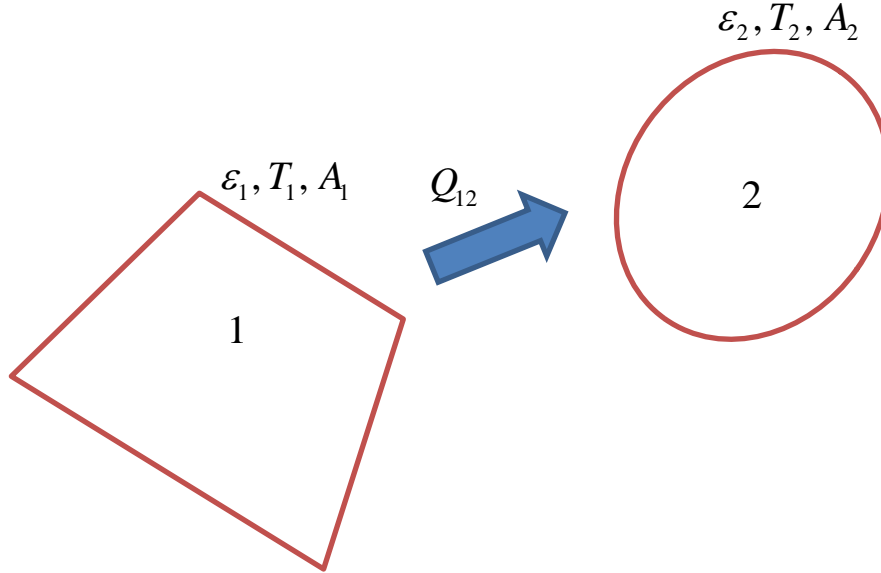
For surface 1, the radiosity $J_1 = \sigma T_1^4$. What is the irradiation? It is the radiosity of surface 2, because all of the radiation leaving surface 2 must fall on surface 1. Thus, $G_1 = \sigma T_2^4$. Therefore, the net heat flux leaving surface 1 at steady state is given by

$$q_1 = J_1 - G_1 = \sigma(T_1^4 - T_2^4)$$

Of course, the net heat flux leaving surface 2 at steady state must be exactly the negative of this result.

$$q_2 = \sigma(T_2^4 - T_1^4)$$

In the case of gray bodies, we must introduce emissivities in the radiant energy fluxes that leave the two surfaces. Also, the two surfaces may not “see” only each other, if they are of finite size.



Consider body 1 with emissivity, absolute surface temperature, and surface area ε_1, T_1 , and A_1 , respectively. It is “seeing” another body 2 with emissivity, absolute surface temperature, and surface area ε_2, T_2 , and A_2 , respectively, as shown schematically in the sketch. Of course, only a part of the surface of body 1 sees a part of the surface of body 2. To accommodate these factors, we write the net rate at which radiation exchange occurs from 1 to 2 as

$$Q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

where F_{12} is called a transfer factor that depends on the emittances of the two surfaces, the geometry, and the fractions of the two surfaces that see each other. If surface 1 is completely surrounded by surface 2, as for example when an object is placed inside a furnace or oven, and $A_1 \ll A_2$, or if surface 2 is black, then we can write $F_{12} \approx \varepsilon_1$ so that

$$Q_{12} = A_1 \varepsilon_1 \sigma (T_1^4 - T_2^4)$$

Radiative Heat Transfer Coefficient

If gray surface 1 is completely surrounded by gray surface 2, and the difference in temperature $\Delta T = (T_1 - T_2)$ is much smaller than either T_1 or T_2 , it is possible to make an approximation. We first expand $T_1^4 - T_2^4$ as follows.

$$T_1^4 - T_2^4 = (T_1^2 + T_2^2)(T_1^2 - T_2^2) = (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$$

In the right side, we can replace $(T_1 + T_2)$ by $2T_m$ where $T_m = (T_1 + T_2)/2$. We also approximate $(T_1^2 + T_2^2)$ by $2T_m^2$. Thus,

$$Q_{12} = A_1 \varepsilon_1 \sigma (T_1^4 - T_2^4) = 4A_1 \varepsilon_1 \sigma T_m^3 (T_1 - T_2) = \boxed{h_r A_1 (T_1 - T_2)}$$

where $h_r = 4 \varepsilon_1 \sigma T_m^3$ is called the radiation heat transfer coefficient.

References

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