Heat Transfer in Flow Past Objects

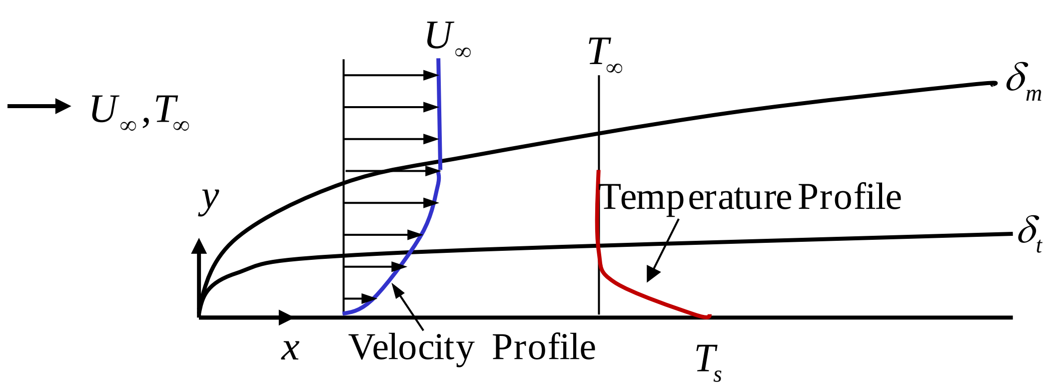
R. Shankar Subramanian
Department of Chemical and Biomolecular Engineering
Clarkson University

Considerable work has been done on predicting heat transfer when a fluid flows past an object. Welty et al. (1) discuss correlations applicable for such flows in Chapter 20 of the textbook. Here, we discuss the most simple situation, namely, heat transfer when a fluid flows over a flat plate.

Flow Over a Flat Plate

External flows occur when the fluid is confined in such a large channel or container that it can be considered practically unbounded in extent when considering heat transfer to or from a stationary solid surface. The simplest case is that of flow over a flat plate that is long and wide. We know that in this situation, for flow at large values of a suitably defined Reynolds number, a momentum boundary layer forms on the surface in which the velocity varies from zero (no slip) at the solid surface to the value in the free stream. Outside the boundary layer, viscous forces are entirely negligible, and potential flow can be assumed to prevail. Potential flow means flow in which the vorticity is zero and viscous forces are neglected. Viscous forces are as important as inertia within the momentum boundary layer.

If the solid surface is maintained at a different temperature from that of the fluid, just like the momentum boundary layer, a thermal boundary layer also forms on the solid surface. It is within this boundary layer that the temperature of the fluid changes from the value at the solid surface to that in the free stream approaching the flat plate. The relative extent of these boundary layers is determined by the value of the Prandtl number of the fluid. A large Prandtl number signifies that momentum is transported more rapidly by molecular means than energy. This implies a thin thermal boundary layer, when compared with the momentum boundary layer. Likewise, a small Prandtl number (as is the case for liquid metals) implies a thicker thermal boundary layer than the momentum boundary layer. A qualitative sketch of both boundary layers, along with typical profiles of velocity and temperature, is given below for a fluid with a large Prandtl number.



In the sketch, T_s is the temperature at the surface of the solid, assumed here to be larger than the temperature in the uniform stream T_∞, that approaches with a uniform velocity U_∞. Note how the velocity approaches U_∞ at the edge of the momentum boundary layer, identified by the symbol δ_m, in a smooth manner, with the slope of the profile approaching zero at the edge of the boundary layer. In a like manner, the temperature of the fluid varies from T_s at the solid surface to T_∞ at the edge of the thermal boundary layer, identified by the symbol δ_t; again, the temperature profile must show a smooth approach to T_∞ with a zero slope at the edge of the thermal boundary layer. Incidentally, the temperature profile is plotted in a different manner in the textbook by Welty et al. (1) in Figure 19.1. While it is fine to plot a temperature difference $\left(T_s - T_\infty\right)$ as the authors do, it is not fine to use the lines with arrows as in the velocity profile. Those lines are used to designate the vector velocity in the velocity distribution. Temperature is not a vector, and the arrows provide a misleading impression in the temperature profile. I recommend plotting temperature profiles as shown in the sketch here.

The most important entity in the heat transfer situation depicted here is the slope of the temperature profile at the solid surface. It is this slope, $\dfrac{\partial T}{\partial y}\left(x,0\right)$, that is crucial in determining the rate at which heat is conducted from the solid into the fluid. We can define a local heat transfer coefficient h as follows.

$$q_s = -k\frac{\partial T}{\partial y}\left(x,0\right) \;=\; h\left(T_s - T_\infty\right)$$

Here, q_s is the heat flux from the solid surface to the fluid. Note that the definition is correct regardless of whether heat flows from the solid to the fluid as in our example, or from a hot fluid to a cold solid. In engineering practice, we are more interested in the average heat transfer coefficient for a plate of length L. If the width of the plate is W, we can write the heat transfer rate from the plate to the fluid at the location x on the solid surface over a region of length dx and width W as

$$dQ = \; q_s\, W\, dx \;=\; h\left(T_s - T_\infty\right) W\, dx$$

from which the total heat transferred over the length of the plate is obtained as

$Q \;=\; \int_0^Q dQ = \int_0^L h\left(T_s - T_\infty\right) W\, dx \;= \left(T_s - T_\infty\right) W \int_0^L h\, dx$ because $\left(T_s - T_\infty\right) W$ is constant over the length of the plate. We also can write $Q = h_{average}\, WL\left(T_s - T_\infty\right)$ so that

$$h_{average} \;= \frac{1}{L}\int_0^L h\, dx$$

and we can define an average Nusselt number as

$$Nu_{average} = \frac{h_{average}\, L}{k}$$

The momentum boundary layer remains laminar until the local Reynolds number $\mathrm{Re}_x = \dfrac{xU_\infty}{\nu}$ reaches a value of approximately of the order 10^5. Beyond that point, transition sets in, and the flow in the boundary layer becomes turbulent when $\mathrm{Re}_x > 3\times10^5$. For a laminar momentum boundary layer, the following correlation for the average Nusselt number is recommended by Mills (2) for ordinary liquids and gases.

$$Nu_{average} = \frac{h_{average} L}{k} = 0.664\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/3}, \qquad \mathrm{Pr} > 0.5$$

Here, $\mathrm{Re}_L = \dfrac{U_\infty L}{\nu}$ where ν is the kinematic viscosity of the fluid.

In the case of liquid metals, a different correlation should be used.

$$Nu_{average} = 1.128\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/2}, \qquad \mathrm{Pr} \ll 1$$

When the flat plate is long and turbulence sets in along the way, the following correlation given by Mills (2) is recommended for the average Nusselt number.

$$Nu_{average} = 0.664\, \mathrm{Re}_{transition}^{1/2}\, \mathrm{Pr}^{1/3} + 0.036\, \mathrm{Re}_L^{0.8}\, \mathrm{Pr}^{0.43}\left[1-\left(\frac{\mathrm{Re}_{transition}}{\mathrm{Re}_L}\right)^{0.8}\right]$$

Mills uses a value $\mathrm{Re}_{transition} = 10^5$ in a worked-out example (4.3). Therefore, that is the value recommended for use in the above correlation. The correlations given by Mills (2) should be used in preference to those in the textbook by Welty et al. (1).

Other external flows

Welty et al. (1) provide correlations for crossflow over cylinders and tube banks, which are useful in making calculations for shell-and-tube heat exchangers. They also provide correlations for flow past spheres. Other useful textbooks to consult are those by Mills (2), Holman (3), and Çengel (4).

References

1. J.R. Welty, G.L. Rorrer, and D.G. Foster, Fundamentals of Momentum, Heat, and Mass Transfer, Sixth Edition, Wiley, New Jersey, 2014.

2. A.F. Mills, *Heat Transfer*, Second Edition, Prentice-Hall, New Jersey, 1999.

3. J. P. Holman, *Heat Transfer*, McGraw-Hill, New York, 2010.

4. Y. A. Çengel, *Heat and Mass Transfer, A Practical Approach*, McGraw-Hill, New York, 2007.