

Some Elementary Aspects of Boundary Layer Theory

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In everyday situations, we encounter fluid flow past objects. Examples occur when we drive a car, swim in water, or go for a bicycle ride. Such flows also occur in numerous engineering applications. We can define a Reynolds number for the flow as $Re = DV/\nu$ where D is a typical dimension of the object, such as the width of a car moving through air, V is the fluid velocity as it approaches the object, and ν is the kinematic viscosity of the fluid. If you calculate this Reynolds number for each of the examples cited earlier in this paragraph, you will find that it is very large. This means that the inertia force in these flows is overwhelming in magnitude when compared with a typical viscous force. Thus, one might expect that completely neglecting the viscous force might be a good first approximation.

If we make the further assumption that the vorticity in the flow is zero, which means that the curl of the velocity field is zero, then the mathematical modeling of such flows becomes very simple. The resulting theoretical framework is called “potential flow” theory. Potential flow theory was used successfully in the nineteenth century to predict a variety of fluid mechanical phenomena. But it was unable to predict the drag that is commonly experienced by objects that are placed in a moving fluid. In fact, it predicted precisely zero drag. One of the physically unrealistic features of potential flow theory is that it is unable to satisfy the no-slip boundary condition at solid surfaces. Everyone recognized that this is related to the reason why potential flow theory was unable to predict drag on objects, but nobody quite knew how to resolve this problem. Then, in 1903, Prandtl developed a powerful and elegant solution that showed how to overcome this limitation of potential flow theory, while still retaining those parts of that theory that were useful. Prandtl’s development came to be known as boundary layer theory.

The key proposal made by Prandtl was that when a fluid flows past an object at high Reynolds number, no matter how small the viscous forces might be in the main flow, they must become large in a thin region right next to a solid surface over which the fluid flows. In this region that Prandtl called the boundary layer, the tangential velocity changes from a rather large value at the edge of the boundary layer to zero at the surface of the solid to satisfy the no-slip boundary condition. Because the boundary layer is thin, this leads to a large velocity gradient in the boundary layer, and therefore, a large viscous force. If one asks “how large,” the answer is: “as large as the inertia force.” In a rough sense, if we define a Reynolds number using the boundary layer thickness as the length scale, instead of a typical dimension of the object, then that new Reynolds number would be of the order of magnitude of unity.

The second important idea put forth by Prandtl was that potential flow theory can be used to obtain the pressure distribution that is impressed on the boundary layer. Because the boundary layer is thin, this pressure distribution can be evaluated along the surface of the object and used as a known entity in the equations describing flow in the boundary layer.

Using Prandtl's boundary layer theory, scientists and engineers were able to predict the drag exerted by fluid flowing past an object quite well. Therefore, this theory now has assumed a central place in fluid mechanics. Also, just as there is a momentum boundary layer in fluid mechanics, we'll see later that there are thermal and concentration boundary layers in certain situations involving heat and mass transfer, respectively.

The principal assumptions made by Prandtl in his proposed boundary layer theory are listed below.

Assumptions

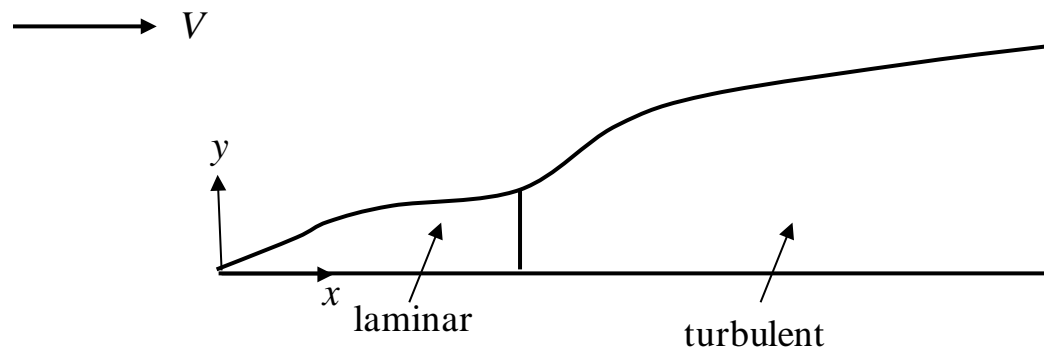
1. When a fluid flows past an object at large values of the Reynolds number, the flow region can be divided into two parts.

(i) Away from the surface of the object, viscous effects can be considered negligible, and potential flow can be assumed.

(ii) In a thin region near the surface of the object, called the boundary layer, viscous effects cannot be neglected, and are as important as inertia.

2. The pressure variation can be calculated from the potential flow solution along the surface of the object, neglecting viscous effects altogether, and assumed to be impressed upon the boundary layer.

A sketch of a typical flow past a long and wide flat plate is shown below. The boundary layer begins by being laminar, but when it reaches a sufficient thickness, flow in the boundary layer becomes turbulent. The turbulent boundary layer grows more rapidly with distance downstream than the laminar boundary layer.



Transition from laminar to turbulent flow in the boundary layer on a flat plate occurs at $Re_x \approx 5 \times 10^5$, where $Re_x = xV / \nu$. Here, ν is the kinematic viscosity of the fluid.

The following approximate estimates can be written for the thickness of laminar and turbulent boundary layers (1).

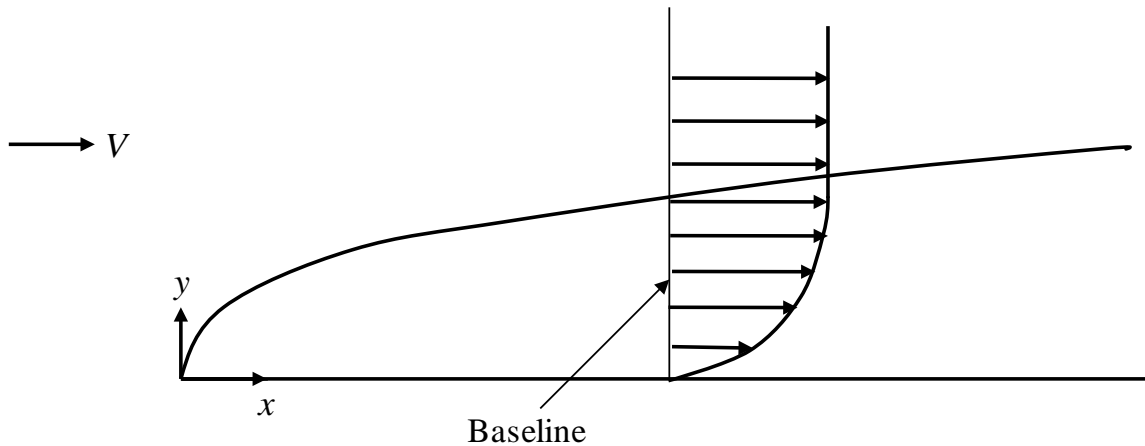
Laminar Boundary Layer

$$\frac{\delta}{x} \sim \frac{5}{\sqrt{\text{Re}_x}}$$

Turbulent Boundary Layer

$$\frac{\delta}{x} \sim \frac{0.16}{(\text{Re}_x)^{1/7}}$$

A typical velocity profile in a laminar boundary layer is displayed below.



There is a lot more to boundary layers than what we have discussed here. For example, in typical flow situations, the boundary layer separates from the surface behind a body, and this creates a region of recirculating eddies in that region. Because of this, the pressure behind the object is much smaller than the pressure on the forward side, so that there is a net pressure force pushing the object in the direction of fluid flow. This makes a major contribution to the drag on an object, the other being viscous drag on the surface. In fact, automobiles and trucks experience significant pressure drag, and a good design of their shapes minimizes its effect. Airplane wings and bodies are “streamlined” to minimize pressure drag. Also, many species of fish have naturally evolved a streamlined shape that minimizes the contribution of pressure drag.

You can learn more about boundary layers from Schlichting (2) and from a series of four videos by A. Shapiro titled “Fluid Dynamics of Drag” Parts I through IV that are available from a web site at MIT. A link to that web site can be found at the course web page.

References

1. F.M. White, Fluid Mechanics, McGraw-Hill, New York, 2011.
2. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York, 1968.