SIMULATION OF PANCAKE ICE DYNAMICS IN A WAVE FIELD

Susan Frankenstein¹, Mark A. Hopkins¹ and Hayley H. Shen²

ABSTRACT
From numerous field observations, it has become well known that pancake ice is ubiquitous in wave dominated polar seas. These strikingly uniform circular floes are consistently found in Antarctic seas during the ice formation season. Their presence has also been reported in the Bering Sea, the Greenland Sea, and in polynyas and leads within pack ice. Pancake ice forms through a combination of thermodynamic growth and mechanical thickening, caused by rafting of floes that is driven by wave motion. This complex growth process is much faster than pure thermodynamic growth and hence may be the main factor responsible for ice edge advance in marginal ice zones. We have developed a dynamic model of pancake ice that combines a three-dimensional discrete element model with wave hydrodynamics. We have used the model to calculate the dependence of open ocean ice drift speeds on wave amplitude. This drift velocity leads to the formation of an ice accumulation at the edge of the fast ice. We have simulated the accumulation of pancake ice floes at a non-reflecting barrier and studied the rate of thickening and the rate of increase of the impact force on the barrier. However, these studies neglected the effects of inertial forces exerted on the pancake ice floes due to fluid accelerations, wave reflection from the ice edge or barrier, and the feedback of the energy dissipation due to collisional dynamics and fluid drag into the wave hydrodynamics. We add these features to the computer model and repeat the previous studies of open ocean drift velocity.

INTRODUCTION
Pancake ice is commonly found in polar regions where waves are present. Lange et al. (1989) described their observation of this phenomenon in the Weddell Sea. Wadhams (personal communication) estimated the areal coverage of pancake ice in the Southern Ocean to be 6 million km². Pancake ice is also common in many marginal ice zones of the northern hemisphere, including the Odden ice tongue in the Greenland Sea, the Okhotsk Sea, and the Bering Sea. Pancake ice forms through a combination of thermodynamic growth and mechanical thickening, caused by rafting of floes driven by wave motion. This complex growth process can thicken the ice cover more rapidly than pure thermodynamic growth and hence may be the main factor responsible for ice edge advance in marginal ice zones. Rafting produces an ice cover with a layered structure.
Fieldwork carried out in the Okhotsk Sea reported rafted ice samples with two to seven layers, each 5–10 cm thick (Toyota, 1998).

Objects floating on the open ocean are subject to oscillating forces that act to accelerate and decelerate their motion due to the cyclic nature of fluid motion within a wave and to the presence of multidirectional waves. The resulting floe motions and floe-floe interactions are not steady. Historically, researchers investigating the motion of ice floes resulting from wave action only included inertial terms relating to the motion of the floes, they did not include those associated with the fluid. They also ignored the advective acceleration terms. In this paper, we correct this oversight and compare the results to the previous work of Hopkins and Shen (2001) investigating the motion and behavior of pancake ice floes subject to a uniform wave field.

**THEORY**

Previous simulations of the dynamics of ice floes in a wave field began with the following one-dimensional equation for the motion of a circular ice floe

\[
\frac{\pi}{4} D^2 h \rho_{\text{ice}} (1 + C_m) \frac{d^2 \bar{x}}{dt^2} = F_g + F_b + F_d + F_c
\]

where \( D \) is the floe diameter, \( h \) is the floe thickness, \( \rho_{\text{ice}} \) is the ice density, and \( C_m \) is the added mass effect due to the oscillatory motion of the floe. The forces on the floe are the buoyancy force \( F_b \), the gravity force \( F_g \), the drag \( F_d \) due to water underneath the floe, and a collisional contact force \( F_c \). The forces on a pair of rafting floes are shown in Figure 1.

![Figure 1: Schematics of the forces acting on floe 1 and floe 2. External forces are only shown on floe 1. Similar forces act on floe 2 also. The interaction force between floe 1 and 2 is from floe-floe collision.](image)

This model without the floe interaction term was first introduced by Rumer et al. (1979) to study ice motion in the Great Lakes. Frankenstein and Shen (1993) included floe interactions and showed that floe collision rates increased rapidly with increasing amplitude. They also showed that drifting of floe agglomerates can be faster than single floes under the same wave conditions. Hopkins and Shen (2001) also used this model to investigate the motion of pancake ice in a plane wave field.

Wherever two floes touch, the overlap is interpreted as a deformation of the floes resulting in a contact force. The contact force has components normal and tangential to the surfaces at the point of contact. The normal axis \( \vec{n} \) is perpendicular to the surface of
each floe. The normal component of the contact force is

$$F^n_n = k_{ne} \delta - k_{nv} \tilde{V}_{1/2} \cdot \tilde{n}$$

(2)

where \(F^n_n\) denotes the normal direction, the superscript \(n\) denotes the current time step, \(k_{ne}\) is the normal contact stiffness, \(\delta\) is the depth of overlap between the floes, \(k_{nv}\) is the normal contact viscosity, and \(\tilde{V}_{1/2}\) is the relative velocity of floe 1 with respect to floe 2 at the point of contact. A value of \(k_{nv}\) near critical damping is used to produce highly inelastic behavior. Tensile forces are not modeled. The tangential axis \(t\) is in the direction of the tangential component of the relative velocity at the point of contact. The incremental change in the tangential force due to friction is proportional to the relative tangential velocity. The tangential force at time \(n\) is

$$\tilde{F}^t_t = \tilde{F}^{t-1}_t - k_{nt} \Delta t (\tilde{V}_{1/2} \cdot \tilde{t}) \tilde{t}$$

(3)

where \(\Delta t\) is the time step and \(k_{nt}\) is the tangential contact stiffness that is set to 60% of \(k_{ne}\). The magnitude of \(k_{nt}\) affects the rate at which the frictional force increases to the Coulomb limit \(\mu F_n\), where \(\mu\) is the friction coefficient. If the tangential force \(F_t\) exceeds the Coulomb limit, the \(x\), \(y\), and \(z\) components of \(F_t\) are scaled such that \(|F_t| = \mu F_n\). The torques on each floe are calculated from the forces and moment arms.

The water drag force \(F_d\) on a floe is given by

$$\tilde{F}_d = -\frac{1}{2} C_d \rho_w A (\tilde{V} - \tilde{V}_w) \tilde{V} - \tilde{V}_w$$

(4)

where \(C_d\) is the drag coefficient, \(\tilde{V}_w\) is the water velocity, \(\rho_w\) is the water density, and \(A = \pi (R_1 + R_2)^2\) is the floe area. The disks are modeled as cylinders with spherical sides such that the floe diameter is \(2(R_1 + R_2)\) and the floe thickness is \(2R_2\). The drag force is separated into components normal and tangential to the flat surface of a floe. The drag coefficient \(C_d\), used in the simulations, was 0.6 for flow normal to the flat surface and 0.06 for flow tangential to the flat surface. The \(x\) and \(z\) components of the water velocity \(U_w\) and \(V_w\) at the floe position \(x\) are

$$U_w = \frac{1}{2} H \omega \cos(kx - \omega t) \quad \text{and} \quad V_w = \frac{1}{2} H \omega \sin(kx - \omega t)$$

(5a,b)

where \(H\) is the wave amplitude, \(k = 2\pi / L\) is the wave number, \(L\) is the wavelength and for deep water the wave frequency, \(\omega = \sqrt{gk}\). Rotational drag \(M_d\) on a floe is given by

$$M_d = -\frac{1}{2} C_d R_1^2 \rho_w A \Omega$$

(6)

where \(\Omega\) is the floe rotational velocity. The three components of the rotational drag are calculated in the body coordinate frame of the floe. The rotational drag coefficient \(C_d\), used in the simulations, was 0.6 for rotation about the \(x\) and \(y\) body axes (in the plane of the floe) and 0.06 for rotation about the \(z\) body axis (normal to the floe). Water drag was applied only to the underwater floes that were in an exposed position. No drag was applied to floes in the interior of a mass of floes. Added mass was included by multiplying the floe mass by \(1 + C_m\) in the equations of motion. The added mass coefficient in the simulations was 0.15. Hydrodynamic lift was not modeled.

Following Hopkins and Shen (2001) the buoyant force and its moment on each floe was calculated by evaluating a numerical surface integral of the hydrostatic pressure on the floe. The hydrostatic pressure on a differential area element \(d\tilde{P}\) was given by

$$d\tilde{P} = -\rho_w g (\eta - z) \tilde{n} dA$$

(7)
where $\mathbf{n}$ is the outward normal to the differential element of area $dA$, $\eta$ is the water surface elevation, and $z$ is the elevation of the differential element. The equation for the water surface is

$$\eta = \frac{1}{2} H \cos(kx - \omega t).$$

(8)

The buoyant force that results from integrating $d\mathbf{P}$ (Eqn. (7)) over the surface of the floe is not the same as the hydrostatic buoyancy force which is defined by Archimede’s Principle as

$$\mathbf{F}_b = \rho_w V_{sub} \mathbf{g}$$

(9)

Because it is impractical to calculate the surface integral for each floe at each time step, a look-up table was created. The components of the buoyancy force and moment were calculated for discrete values of 4 independent variables; the depth of the floe center below the actual water surface, the angle between the body $z$ axis (floe normal) and the global $z$ axis, the azimuthal angle formed by the projection of the body $z$ axis on the global $xy$ plane and the global $x$ axis, and finally the inclination angle of the wave surface. The 4 variables were discretized respectively into 30, 18, 36, and 10 intervals. A quadri-variate interpolation scheme was used to interpolate between discrete functional values. Five dependent variables are calculated, the $x$ and $z$ components of the buoyant force and the $x$, $y$, and $z$ components of the moment.

Except for the drag force, Eqn. (1) fails to consider all of the effects associated with the fact that both the water and the floe move. Following the work of Newman (1977) and Sarpkaya and Isaacson (1981), the forces on a floe when both the floe and fluid are moving are

$$\left(\rho_{ice} V_{ice} + \rho_{ice} V_{sub} C_m\right) \frac{d\mathbf{V}}{dt} = \mathbf{F}_d + \mathbf{F}_c + \mathbf{F}_g + \mathbf{F}_b + \rho_w V_{sub} \mathbf{g}$$

$$+ \rho_w V_{sub} \left(1 + C_m\right) \left[ \frac{\partial \mathbf{V}}{\partial t} + ((\mathbf{V}_w - \mathbf{V}) \cdot \nabla) \mathbf{V}_w \right]$$

(10)

where $\mathbf{F}_b$ is the same as in Eqn. (9), $V_{sub}$ is the submerged floe volume and $V_{ice}$ is the total ice floe volume. The left hand sides of Eqn. (1) and (10) vary slightly in the added mass term. In Eqn. (1), the added mass is multiplied by the total floe mass while in Eqn. (10) it is multiplied by only the submerged mass of the floe. The additional terms on the right hand side of Eqn. (10) compared to Eqn. (1) originate from integrating the dynamic pressure field as a result of the moving fluid on a floe. In the $x$- and $z$-direction they are

$$\text{New} = \rho_w V_{sub} C_m \frac{\partial U_w}{\partial t} + \rho_w V_{sub} \left(1 + C_m\right) \left[ (U_w - U) \frac{\partial U_w}{\partial x} + (W_w - W) \frac{\partial W_w}{\partial z} \right] x - \text{dir}$$

$$\text{New} = \rho_w V_{sub} g + \rho_w V_{sub} \left(1 + C_m\right) \left[ \frac{\partial W_w}{\partial t} + (U_w - U) \frac{\partial W_w}{\partial x} + (W_w - W) \frac{\partial W_w}{\partial z} \right] z - \text{dir}$$

(11)

Unlike the other forces, the new terms are treated as point sources. In other words, they represent the additional inertial forces caused by the fluid moving around a non-stationary object. It is interesting to note that integration of the hydrodynamic pressure force (Eqn. (7)) in the $x$-direction is equivalent to the $\rho_w V_{sub} \partial U_w / \partial t$ term in Eqn. (10).
RESULTS
A set of simulations was performed with the computer model described above. Each simulation began with a single layer of floes distributed uniformly over the water surface with an areal concentration of 50%. The waves moved in the longitudinal or x-direction. In all simulations the lateral boundaries of the domain (y-direction) were periodic. Periodic boundaries are routinely used in discrete element simulations to eliminate solid boundaries (see for example Hopkins and Louge, 1991). In the absence of solid boundaries, the floes on the right boundary interact with the periodic image of the floes on the left boundary and vice versa. The z-axis was vertical. In order to study the drift velocities, the domain end boundaries (x-direction) were also periodic, creating what amounts to an infinite domain. The floe-floe dynamics were turned off and the floes were allowed to drift freely with the waves. The parameters used in the simulations are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>$L$</td>
<td>100 m</td>
</tr>
<tr>
<td>Wave height</td>
<td>$H$</td>
<td>2, 3, 4, 5 m</td>
</tr>
<tr>
<td>Domain length</td>
<td></td>
<td>600 m</td>
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<tr>
<td>Domain width</td>
<td></td>
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<tr>
<td>Floe thickness</td>
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<td>167 mm</td>
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<tr>
<td>Floe diameter</td>
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</tr>
<tr>
<td>Water density</td>
<td>$\rho_w$</td>
<td>1010 kg m$^{-3}$</td>
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<tr>
<td>Normal contact stiffness</td>
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<tr>
<td>Floe surface friction</td>
<td>$\mu$</td>
<td>0.35</td>
</tr>
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</table>

The duration of each simulation was 300 s. The drift velocity was calculated at regular time intervals by sorting the floes into bins depending on the current phase angle of the location of their centers. The wavelength was divided into 100 bins. The x-component of velocity of the floes in each bin was averaged. Figure 2 shows the difference between the average drift velocity and the water speed $U_w$ at the surface (5a) as a function of phase angle and for both the original and new cases for a 5.0 m amplitude wave. The results shown are averaged over a 20 s period at the end of the simulation well after the drift velocity has equilibrated.

The “Old” case in Figure 2 is a plot of Eqn. (1) while the “New” cases are plots of Eqn. (10). The different values of “New” indicate the percent that the terms shown in Eqn. (11) are multiplied by before being incorporated into Eqn. (10). Thus, the only difference between the “Old” and “New, 0%” plots are due strictly to discrepancies with the left hand sides of Eqn.s (1) and (10) respectively. This change in itself causes the floe motion to be damped. Increasing the effect of the new force results in further floe motion damping. The averaging scheme takes into consideration not only the number of floes, but also their relative position along the wave. Thus, even though from Figure 2 it appears that the floe motion reverses itself relative to the wave direction as more of the
new force is added, few floes are found at these positions so that the overall average remains positive.

Figure 3 shows the newly calculated drift velocity as a function of wave amplitude. Two peaks are evident, one near the trough of the wave, and one near the crest for all cases. This was not observed in the original simulations (Hopkins and Shen, 2001). The dip in the floe drift occurs as the wave transitions from the crest to trough and visa versa. The size of this dip decreases as the wave amplitude decreases. Hopkins and Shen (2001) found that floe drift was greatest at the wave trough and smallest at the wave crest. Another difference between the original and new results is the smoothness of the curves. The new force terms lead to more chaotic motion. Again, even though it appears from Figure 3 that the average floe motion should be negative or approximately zero, more floes are found near the peak and trough of the wave so that the position weighted average drift velocity is positive, i.e., in the direction of the wave.

CONCLUSIONS
Based on these preliminary results, the additional terms associated with the fluid’s inertia in Eqn. (10) compared to Eqn. (1) do influence floe drift and that this influence
changes depending on the floe’s position along the wave as well as the wave’s amplitude.

To date, only the influence of wave amplitude has been investigated. The next steps will be to allow floe-floe interactions then see how the floes aggregate in front of a fixed barrier. In the latter case, we will include the influence of wave reflection from the barrier as well as damping of the wave as the floes amass. The damping of wave energy translates to a decrease in amplitude as has been observed in the field.

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REFERENCES


