3D NUMERICAL SIMULATION OF LEVEL ICE-STRUCTURE INTERACTION

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ABSTRACT
This paper presents the development of a 3D numerical simulation model for level ice-structure interaction. The finite-difference methodology of Wilkins is used. Different failure criteria (Coulomb-Mohr, Drucker-Prager and Schulson) are considered. The analysis shows the excellence of the Schulson criterion. The results of the numerical experiments are discussed.

INTRODUCTION
Several authors have considered numerical simulations of ice-structure interactions (e.g. Kärnä, 1995; Shkhinek et al., 1999; Kärnä et al., 2001). However, all these studies examined only 2D solutions either in plane or normal to plane. Kärnä et al. (2001) tried to include some corrections to the 2D solution in order to achieve an approximation of the 3D phenomena, but this result was based on several assumptions. Most of these assumptions have a physical background, but some coefficients were not well-founded.

Another important aspect of the ice-structure interaction is the mathematical model for ice failure. The literature comprises several models such as Coulomb-Mohr, Drucker-Prager, and Schulson (Schulson et al., 1991; Smith and Schulson, 1993). The comparison of different 3D solutions and failure models with experimental data obtained by Kärnä (1995), gives the opportunity of choosing the best theoretical model for the numerical simulation.

STATEMENT OF THE PROBLEM
Assume an infinite ice sheet with thickness \( h \) (sufficient to avoid buckling) moves with velocity \( U_\infty \) against a structure (vertical, immovable, absolutely rigid wall) of width \( L \) (Fig. 1). The ice sheet covers the surface described by \( X > 0, -\infty < Y < \infty \). As the structure has a limited width and the ice a limited thickness, the phenomena should be considered as 3D. Further we assume that the sheet hits the wall at time \( t = 0 \). The task is now to consider peculiarities of the stress/strain field in the 3D solution and to compare the excellence of the different mathematical ice failure models (Coulomb-

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THE CONSTITUTIVE EQUATIONS

Two systems of coordinates are used in the computations. The global one \((X,Y,Z)\) is related to the entire ice field. The local system is related to the elements that represent parts of the ice field. Let us assume that ice failure can develop both by shear and by tension.

Figure 1: Sketch of the computational set-up.

Criteria of shear failure

It is assumed that the elastic-plastic model describes the failure of this type. It proposed also that ice velocity is sufficiently high and creep does not exist. Initially the material is considered as elastic. As soon as the failure criterion is reached in any point of the material, the ice properties in such a point is set to a residual state with a reduced strength. The main equations for the 3D phenomenon may be written in the following form \((i,j = 1, 2, 3)\):

\[
\begin{align*}
\rho V &= \rho_0 V_0 \quad (1a) \\
\rho \frac{dU_i}{dt} &= \frac{\partial S_{ij}}{\partial x_j} - \frac{\partial P}{\partial x_i} \quad (1b) \\
\frac{DS_{ij}}{Dt} &= 2G \left( e_{ij} - \frac{1}{3} e_{kk} \delta_{ij} - \dot{\lambda} S_{ij} \right) \quad (1c) \\
\frac{dP}{dt} &= -K \left( e_{kk} - 2\dot{\lambda} \dot{\tau} \right) \quad (1d)
\end{align*}
\]

Summation is made over the recurring indexes and the following notation is used:\n
\(x_i \equiv X, x_2 \equiv Y, x_3 \equiv Z\). \(\rho, \rho_0\) and \(V, V_0\) are the ice densities and volume in the current and the initial states, respectively. \(U_i\) is the projection of the velocity vector on the \(x_i\)-axis of the global system of coordinates and \(t\) is time. \(G\) and \(K\) are the shear and bulk modulus, and \(\frac{D}{Dt}\) is the Jaumanns derivative. \(e_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)\) are the components of the tensor of the velocity of deformation, and \(\dot{\lambda}\) is the velocity of dilatancy (assumed equal to zero in the numerical experiments). \(\dot{\tau}^2 = \frac{3}{8} S_{ij} S_{ij}\) is the intensity of the shear stress and \(\dot{\lambda}\) is the factor to be determined from the conditions that stresses reach on the ultimate strength surface (otherwise \(\dot{\lambda} = 0\)). The components of the stress tensor are considered as a sum of the components of the spherical stress tensor and the deviatoric stress tensor, and read \(\sigma_{ij} = -P \delta_{ij} + S_{ij}\) where \(P = -\sigma_{kk} / 3\) is
the hydrostatic pressure and $\delta_{ij}$ is the Kroneker's symbol.

Three different strength criteria were considered: Coulomb-Mohr, Drucker-Prager and Schulson (Schulson et al., 1991; Smith and Schulson, 1993). The main equations of these criteria follows next:

**The Coulomb-Mohr criterion:**

$$\sigma_i' = -R_c + K_1 \sigma_i$$
$$K_1 = (1 + \sin \varphi)/(1 - \sin \varphi)$$

(2)

where $\sigma_i$ is the maximum principle stress (compressive stresses are negative), $\sigma_i'$ is the minimum principle stress located on the failure surface. $R_c$ is the uniaxial compressive strength and $\varphi$ is the angle of internal friction.

**The generalized Coulomb-Mohr criterion** is written in the form suggested by Drucker-Prager. It takes into account the strength dependence on all the principal stresses:

$$\sqrt{3I_2} = Y_i + Y_2 P$$

(3)

here $I_2 = \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)/6$ is the ultimate value of the second invariant of the deviatoric stress tensor, and $Y_2 = 3(tg^2 \alpha - 1)/(tg^2 \alpha + 2)$, $Y_i = R_c (1 - Y_2 / 3), \alpha = 45^\circ + \varphi / 2$ are some functions. Eq. (3) turns into Eq. (2) in the special case when $\sigma_2 = \sigma_1$.

**The Schulson criterion** was obtained from analysis of his physical 2D experiments. It was shown that depending on the correlation between some parameters $R_A$ and $R_T$, two regimes could develop during compression along the crystals. The border between the two regimes was determined from the equation $R_A = R_T$ where $R_A = \sigma_1 / \sigma_3$ ($\sigma_1$ and $\sigma_3$ are the minimum and the maximum principle stresses in the plane of loading). The parameter $R_T$ can be written in the form:

$$R_T = (K_2 - R_c)/(R_c + K_1 K_2)$$

where $K_1$ is determined by Eq. (2). According to Smith and Schulson (1993), $K_2$ takes the form:

$$K_2 = 6 \sigma_t / (1 - 2 \nu)$$

where $\sigma_t$ is the tensile strength and $\nu$ is the Poisson's ratio. The parameter $R_t = 0.2$ for $\sigma_t = 0.2 R_c$, $\varphi = 30^\circ$ and $\nu = 0.3$. This value correlates with the results of Smith and Schulson (1993) who found this parameter to be in the range of 0.2–0.1 if the temperature falls in the range −10 °C to −40 °C.

If $0 < R_A < R_T$, then the strength increases if the lateral pressure increases, and the strength criterion can be written in the form of Eq. (2). If $R_A > R_T$, then an increase of the lateral pressure leads to a strength decrease and the strength criterion reads:

$$\sigma_i' = -K_2 - \sigma_i$$

(4)

If the maximum stress is oriented along the axis of columns and the lateral loading is normal to them, then the strength does not depend on the lateral pressure (Smith and Schulson, 1993; Frederking, 1977).
Criteria of the tensile failure. It was proposed for the Coulomb-Mohr and Schulson models that tensile failure arises if the maximum principle stress reaches the tensile strength \( \sigma_t \). For the Drucker-Prager model the criterion of the tensile failure was used in the form \( P = -\sigma_t / 3 \). As soon as the criterion of the tensile failure is reached, the maximum principle stress is assumed to be zero in this point.

Initial and boundary conditions
Initial conditions:

\[
U_x = U_y = U_z = 0, \quad \sigma_{ij} = 0
\]

Boundary conditions at the ice-structure contact surface:
Sliding conditions without friction are used in the present solution, but the model allows the use of any other conditions.

Boundary conditions on the quasi-infinite surfaces (\( X = \infty, Y = \infty \)):
As the real infinite area of the ice sheet can not be considered, only the quasi-infinite area is investigated. This means that we exclude any wave reflection from boundary surfaces located at \( X = X_m, Y = Y_m \). This condition can be written as (Lysmer and Kuhlemeyer, 1969):

\[
\sigma_n = -\rho c U_n, \quad \sigma_\tau = -\rho b U_\tau
\]

where \( c \) and \( b \) are velocities of longitudinal and shear elastic waves. \( \sigma_n, \sigma_\tau \) are the normal and tangential components of the stress tensor, and \( U_n, U_\tau \) are the particle velocity vector on the boundary surfaces.

THE COMPUTATIONAL ALGORITHM
The Lagrangian finite difference net was used for the integration of the equations. The volume of ice was divided into cells. These cells do not overlap each other and there are no gaps between them. The apexes of the cells are the nodes of the net. The hexahedron cells that were used in the Wilkins method (Wilkins et al., 1973) were additionally divided into tetrahedron elements (Fig. 2). The components of the velocity are referred to the nodes of the net at the time that corresponds to the half time step \( (t + \Delta t / 2) \). The ice density and the components of the tensors of strain rates and stresses are referred to the cell center and determined at the time corresponding to the whole time step \( (t + \Delta t) \).

Figure 2: The cells for computation. The hexahedron cell and its division into tetrahedron elements.

The finite-difference scheme for integration of the system of equations can be received as a result of replacing the partial derivatives in space by integrals along the contour of
the corresponding elements and use of the theorem about the mean value. The time
derivatives are approximated by the central differences. The equations of motion for any
node of the finite-difference net can be written in the form:

\[ M \frac{\partial U_i}{\partial t} = F_I + f_i \]  

(5)

where \( M \) is the mass concentrated in the node and equal to a fourth of the mass of all
elements surrounding this node. \( F_I \) is the projection on the axis \( i \) of the internal forces
acting on this node, and \( f_i \) is the projection of the external forces (these forces act only
on the boundary nodes). The force \( F_{ij}^l \) that acts on the node \( l \) from each tetrahedron
may be determined by the formula:

\[ F_{ij}^l = \sigma_{ij} n_{ij}^{(k)} S^{(k)}; \quad k = 1, \ldots, 4 \]

where \( \sigma_{ij} \) is the stress tensor, \( S^{(k)} \) is the area of the facet \( k \) of the tetrahedron, \( n^{(k)} \) is the
external normal. The total force is obtained by summation over all elements surrounding
the node. The stress-strain field is determined from the integration of the system of the
constitutive equations. A linear approximation of the particle velocity along the
coordinates within the cell is assumed. Finally the formula for the strain rate calculation
takes the form:

\[ e_{ij} = -\frac{1}{6V} \sum_{k=1}^{4} \left( U_{ij}^{(k)} n_{ij}^{(k)} + U_{ij}^{(k)} n_{ij}^{(k)} \right) S^{(k)} \]  

(6)

The following steps for the computations were used:

i) The components of the increments of the particle velocity at time \( (t + \Delta t / 2) \) are
determined for each node from the stress field in the surrounding points (at time \( t \)):

\[ U_{ij}^{(t)}(t + \Delta t / 2) - U_{ij}^{(t)}(t - \Delta t / 2) = \frac{\Delta t}{M^{(t)}} \left( F_{ij}^{(t)}(t) + f_{ij}^{(t)}(t) \right) \]  

(7)

ii) The new position of the nodes of the Lagrangian net are determined by the formula:

\[ x_i^{(t)}(t + \Delta t) = x_i^{(t)}(t) + \Delta t U_i^{(t)}(t + \Delta t / 2) \]

iii) The elastic components \( \sigma^e \) (for \( \lambda = 0 \)) of the stress field and the strain rate are
determined from Eqs. (1c–1d) and Eq. (6).

iv) The elastic stress field \( \sigma^e \) is analysed in the following order:

• First the possibility of tensile failure in the cell is checked. If tensile failure
  occurs, then the stress field in this cell is corrected. The corrected stress field is
  checked again, and the possibility for shear failure is examined. If shear failure
  may take place, then an additional correction is made.
  • If tensile failure does not occur, then the possibility of shear failure is considered.
  If shear failure takes place, then the corresponding correction is made and the next
time step is done after corrections. The particle velocities are determined (item i))
  and the whole algorithm is repeated.

**SOME RESULTS FROM THE NUMERICAL EXPERIMENTS**

Numerical experiments were carried out to compare results predicted by the different
models and to analyse the effect of the major model parameters on the load.

**The Coulomb-Mohr versus Drucker-Prager models**

A comparison of calculation results from these models shows that the difference is just
minor, but the Coulomb-Mohr model needs significantly more computations because
the principle stresses have to be determined for each time step. This procedure requires additional computational time but this is not justified by the increased accuracy of the results. Therefore we continue with just using the Drucker-Prager and Schulson models.

The Drucker-Prager versus Schulson models
The pressures predicted by these models differ depending on the ice velocity, residual strength, aspect ratio etc. Especially a noticeable difference can be seen in the failure pattern. The Schulson model predicts formation of some failure curves ahead of the structure surface. The dimension of the failure zone depends on the ice properties and velocity. An increase of the velocity leads to a decrease in the extension of the failure zone. The similar phenomenon was observed in Kärnä’s experiments (Kärnä, 1995). On the contrary, the Drucker-Prager model predicts the failure progress along the contact surface in all situations. Apparently the Drucker-Prager model does not provide the prediction of the failure pattern.

Influence of ice velocity
It was shown in the 2D solutions (Kärnä et al., 2001) that pressure versus time comprises two parts: the first is connected to the initial hit of the ice sheet with the structure, and the second one with the structure penetration in the ice sheet. This result is confirmed in the 3D solution. In particular this can be seen in Fig. 3. The non-dimensional pressure (pressure referred to the unconfined strength) time series in the different points of the Y-axis (see Fig. 1) are depicted in this figure. Initially the pressure jump induced by the first hit is distributed evenly along the contact area. It can be seen in particular in this figure that when the time is about zero the pressure ($\approx 0.25$) is the same in all $Y/L$. Later, due to the structure penetration in the ice sheet, the pressure is distributed unevenly with concentration at the corners. The level of the first part of the pressure is directly proportional to the ice velocity, density and modulus of deformation. On the contrary, the level of the second part depends on the ice strength $R_c$, residual strength, $\phi$, $\sigma$, and $\nu$. The relationship between these parts depends on the ice velocity and properties. If $R_c, \phi \approx 30^\circ$ then at $U_\infty > 0.7$ m/s the first part predominates.

Figure 3: The non-dimensional pressure distribution along the Y-axis of the contact surface at different times ($L = 20$ m, $h = 2$ m, $U_\infty = 0.1$ m/s).
The stress concentration
It can be seen in Fig. 3. that the pressure at the corner (stress concentration) develops gradually. At the time when this stress reaches maximum, the average non-dimensional pressure (the load divided by the formal contact area $h \times L$ and $R_c$) is not maximal. The last is maximal later when stresses distributed more evenly along the contact surface. This can be seen in Fig. 4 with input data as shown in Fig. 3. The maximal load in this experiment corresponds to the time 35.65 ms.

This means that the maximal stress concentration does not control the maximal load.

CONCLUSIONS
A 3D solution of level ice-structure interaction was developed and numerical experiments carried out. The major conclusions of the study are:

1. It was shown that the Coulomb-Mohr failure criterion has no advantage in accuracy compared with the Drucker-Prager, besides it needs significantly more computational time for 3D calculations.

2. The failure patterns in physical experiments correlate best with the pattern predicted by the Schulson model.

3. The load comprises two parts: One part relates to the first hit the structure by the ice, the second part is connected to the structure penetrating the ice. The correlation between these parts depends mostly on the ice speed - the higher the speed, the more important the first part is.

4. Although the stress concentration near the corners may exceed significantly the pressure in other points of the contact surface, the maximal global load occurs when the concentration of pressure is not at the maximum, but pressure rather distributed more evenly along the contact surface. So maximal stress concentration do not coincide in time with the maximal global load

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