DATA ASSIMILATION IN RIVER ICE FORECASTING

Steven F. Daly

ABSTRACT
This study presents a state-space model for forecasting ice conditions and the resulting stages in rivers. The model incorporates a hydraulic component, a thermal and ice transport component, and an ice-cover progression component. The Kalman filter procedure is used to update the model with observed stages and observed position of the upstream leading edge of the ice cover. The model thereby arrives at an efficient and optimal estimate of the river ice and hydraulic conditions. The state-space model can also recursively estimate the effective channel roughness using the augmented Kalman filter procedure to account for changes in the channel roughness produced by the river ice cover and other effects. By way of an example, the state-space model is applied to the Missouri River downstream of Oahe Dam, located in Pierre, South Dakota, USA. Outflow from the dam, which is used for peaking power production, can vary between 0 and 55,000 cfs in a matter of minutes to meet the demands of the electric-power grid.

INTRODUCTION
In this study a numerical model with several components is used to simulate the rapidly changing stage, discharges, and ice conditions that occur in the Missouri River. The components are a hydraulic model, which estimates the water surface elevations and flows in the river system; a thermal and ice transport model, which estimates the river water temperatures, the frazil ice and surface ice concentrations, and the surface ice thickness; and an ice progression model, which estimates the extent and thickness of any stationary ice covers. Due to limitations of space, only the hydraulic component will be discussed here, and we must further limit the discussion to estimating changes in the channel conveyance from assimilated stage and ice cover extent data.

The hydraulic model is used to estimate the flow stages and discharges for a channel whose ice cover changes dynamically. The basic continuity and momentum equations describing one-dimensional, unsteady flow in channels with a floodplain were developed for both open-water and ice-covered flow. The continuity and momentum equations are solved using the four-point, implicit finite-difference scheme. As is usual, the channel geometric properties were described at only a finite number of discrete cross sections that segments the river into a series of subreaches. An additional problem is presented by the fact that the ice cover extent along the channel may be continuously

1 US Army Corps of Engineers ERDC-CRREL, 72 Lyme Rd., Hanover, NH
increasing or decreasing with time. A conflict arises with the description of the river channel using discrete cross sections that cause abrupt changes in the results each time the ice cover extent passes through a cross section. To overcome this conflict, a means of interpolating the channel properties between the ice-covered and open-water geometries was developed to smooth the changes as the ice cover advances or retreats past a cross section. This interpolation requires two new variables per cross section: $\Sigma u_{sj}$, which describes the ice cover extent immediately upstream of cross section number $j$, and $\Sigma ds_j$, which describes the ice cover extent immediately downstream of $j$. The rate of ice cover progression is calculated using the straightforward approach originally presented by Lal and Shen (1993). In open water the geometric properties of each cross section need only be determined once, then used for both the upstream and downstream subreach calculations. In the ice-covered case the geometric properties need only be determined once but may need to be modified separately for downstream or upstream subreach calculations. This is done as follows. Let $\Delta u_{ic}$ be the distance that the ice cover extends upstream of cross section $I$ which has a subreach length $\Delta x_i$. Then the following two variables associated with each cross section can be defined:

$$\Sigma u_{sj} = \frac{\Delta u_{ic}}{\Delta x_i} \text{ if } 0 \leq \frac{\Delta u_{ic}}{\Delta x_i} \leq 0.5 \text{; } \Sigma u_{sj} = 0.5 \text{ if } 1 \geq \frac{\Delta u_{ic}}{\Delta x_i} > 0.5 \text{.} \quad (1)$$

and

$$\Sigma ds_j = 0 \text{ if } 0 \leq \frac{\Delta u_{ic+1}}{\Delta x_{i+1}} \leq 0.5 \text{; } \Sigma ds_j = \frac{\Delta u_{ic+1}}{\Delta x_{i+1}} - 0.5 \text{ if } 1 \geq \frac{\Delta u_{ic+1}}{\Delta x_{i+1}} > 0.5 \text{.} \quad (2)$$

Let $R_i$ be any geometric variable at section $i$ determined though the use of the look-up table. Then let $\bar{R}_i$ be the value of the variable weighted for the ice cover presence. The value of $\bar{R}_i$ for use in determining the hydraulic properties of the flow reach defined by sections $i$ and $i+1$ can be estimated as

$$\bar{R}_i = 2\left[ (0.5 - \Sigma ds_j) R_{oi} + \Sigma ds_j R_{ii} \right] \quad (3)$$

where $R_{oi}$ is the open-water value of $R_i$, and $R_{ii}$ is the ice-covered value. The value of $\bar{R}_i$ for use in determining the hydraulic properties of the flow reach defined by section $i - 1$ and $i$ can be estimated as

$$\bar{R}_i = 2\left[ (0.5 - \Sigma u_{sj}) R_{oi} + \Sigma u_{sj} R_{ii} \right] \quad (4)$$

The effective channel conveyance at a section may vary with time because of changes in the cross-sectional area or through changes in the effective hydraulic roughness of the channel bed or the river ice cover. For example, sediment transport may both change the cross-sectional area and alter the effective roughness of a section. In this study the concern is with changes in the effective roughness of the ice cover with time (Shen and Yapa, 1986). In this study a conveyance factor, $C_v$, is proposed that is applied to the effective channel conveyance as shown in the following equation:

$$\bar{K}_i = C_v 2\left[ (0.5 - \Sigma u_{sj}) K_{oi} + \Sigma u_{sj} K_{ii} \right] \quad (5)$$

where $K_{oi}$ and $K_{ii}$ are the open-water and ice-cover channel conveyances, respectively. $C_v$ can also be regarded as the inverse of the hydraulic roughness factor that modifies the composite Manning’s roughness coefficient of a section. As the effective channel roughness decreases, for example through smoothing of the river ice cover, $C_v$ increases.
DATA ASSIMILATION

The state vector at time \( n + 1 \) is \( \mathbf{x}^{n+1} \). It contains flows, stages, ice extents, and conveyance factors as follows:

\[
\mathbf{x}^{n+1} = \left[ Q^{n+1}_1, H^{n+1}_1, \ldots, Q^{n+1}_i, H^{n+1}_i, \Sigma d^{n+1}_1, \ldots, \Sigma d^{n+1}_k, C^{n+1}_1, \ldots, C^{n+1}_k \right]
\]  

(6)

The elements of \( \mathbf{X} \) are random variables with an associated Gaussian probability distribution function. The number of cross sections used in the simulation determines the number of elements in \( \mathbf{X} \). There are separate elements representing the stage, discharge, the downstream ice extent, the upstream ice extent, and the conveyance factors for each section. Altogether there are \( 4n - 2 \) elements in \( \mathbf{X} \), where \( n \) is the total number of sections.

The system model can be written as

\[
f_\epsilon \left( \mathbf{x}^{n+1}, \mathbf{x}^n, \mathbf{u}^{n+1} \right) = \Gamma^n \mathbf{w}^n
\]

(7)

where \( \mathbf{u}^{n+1} \) is the vector of input data. All the elements of \( \mathbf{x}^{n+1} \), \( \mathbf{x}^n \), and \( \mathbf{u}^{n+1} \) are random functions. \( \hat{\mathbf{x}}^{n+1} \), \( \hat{\mathbf{x}}^n \), and \( \hat{\mathbf{u}}^{n+1} \) are the best estimates of \( \mathbf{x}^{n+1} \), \( \mathbf{x}^n \), and \( \mathbf{u}^{n+1} \), defined by the expectation operator, \( E \). \( \hat{\mathbf{x}}^n \) in Eq. (7) is required to specify a system noise value for each equation in \( f_\epsilon \). The system equation can be expanded in a Taylor’s series and then restated as

\[
\mathbf{x}^{n+1} = \Phi^n \mathbf{x}^n + \Lambda^n \mathbf{u}^{n+1} + \mathbf{N}^n + \Gamma^n \mathbf{w}^n
\]

(8)

where \( \mathbf{x}^n \) = column vector of the state variables at time \( n \); \( \Phi^n \) = state transition matrix; \( \Lambda^n \) = input coupling matrix; \( \mathbf{N} \) = matrix containing elements unaffected by the expectation operator; \( \mathbf{w} \) = zero mean additive model error term, which is uncorrelated (or “white”) in time.

The elements of the state vector are random elements with Gaussian probability distribution functions (pdf’s). The best estimate or expected value of the state vector at time \( n \) is defined as \( \hat{\mathbf{x}}^n \). \( \hat{\mathbf{x}}^n = E \left\{ \mathbf{x}^n \right\} \) where \( E \) is the expectation operator. The evolution of the mean of the distribution can be found, to first-order accuracy, by solution of the system of equations

\[
E \left\{ f_\epsilon \left( \mathbf{x}^{n+1}, \mathbf{x}^n, \mathbf{u}^{n+1} \right) \right\} = E \left\{ f_\epsilon \left( \hat{\mathbf{x}}^{n+1}, \hat{\mathbf{x}}^n, \hat{\mathbf{u}}^{n+1} \right) \right\} = 0
\]

(9)

Solving the set of equations \( f_\epsilon \) for the unknown variables in \( \hat{\mathbf{x}}^{n+1} \) provides an estimate of the mean of the pdf’s of the random elements of \( \mathbf{x}^{n+1} \). The equations included in \( f \) are the finite difference form of the governing equations and are solved using the Newton–Raphson procedure.

The covariance of the state variables at time \( n + 1 \) is defined as

\[
\mathbf{P}^{n+1} = E \left\{ \left[ \mathbf{x}^{n+1} - E \left\{ \mathbf{x}^{n+1} \right\} \right] \left[ \mathbf{x}^{n+1} - E \left\{ \mathbf{x}^{n+1} \right\} \right]^T \right\}
\]

(10)

Substituting the state equations for \( \mathbf{x}^{n+1} \), the covariance becomes

\[
\mathbf{P}^{n+1} = \Phi^n \mathbf{P}^n \Phi^n + \Lambda^n \mathbf{P}^n \Lambda^n + \Gamma^n \mathbf{Q} \Gamma^n
\]

(11)

where \( \mathbf{P}_U \) is the covariance matrix associated with the input vector \( \mathbf{U} \). The model input is assumed to be corrupted by white noise with zero mean.

The relationship between the observations and the state variables can be expressed as

\[
\mathbf{z}^{n+1} = \mathbf{H}^{n+1} \mathbf{x}^{n+1} + \mathbf{v}^{n+1}
\]

(12)
where \( z^{n+1} \) contains the observations made at time \( n \), \( H^{n+1} \) is the observation matrix that relates the observations to the state variables, and \( v \) is the measurement noise vector. It is assumed that \( \mathbf{E}[v] = 0 \) and that \( \text{cov}(v_j, v_k) = \mathbf{E}[v_j v_k^T] = R \delta_{ij} \), where \( \delta_{ij} \) is the Kronecker operator. Typically there are several observations of stage and an observation of the ice cover extent. The observation of the leading edge of the ice cover must be converted into observations of the upstream and downstream ice extent at each section. This produces \( 2n - 2 \) ice observations. Let \( J \) be the number of stage observations. \( z \) is then a vector \( J + 2n - 2 \) elements in length. The matrix \( H \) is \( J + 2n - 2 \) by \( 4n - 2 \) in size and is composed of elements that are either ones or zeros.

There are two means of estimating the state variables: through the solution of the system equation or through observation. The Kalman gain is the procedure by which these two methods of estimation are reconciled. Let \( \hat{X}^{n+1}(-) \) be the a priori (prior to the observation) system estimate and \( P^{n+1}(-) \) be the a priori covariance estimate. Following the discussion of Grewal and Andrews (1993) an updated estimate \( \hat{X}^{n+1}(+) \) is sought that is a linear function of the a priori estimate, \( \hat{X}^{n+1}(-) \), and the observation, \( z^{n+1} \). In the present case the Kalman gain, \( K^{n+1} \) is

\[
K^{n+1} = P^{n+1}(-)H^T \left[ HP^{n+1}(-)H^T + R \right]^{-1}
\]

The system estimate update and covariance update are

\[
\hat{X}^{n+1}(+) = \hat{X}^{n+1}(-) + K^{n+1} \left[ z^{n+1} - H^{n+1} \hat{X}^{n+1}(-) \right]
\]

\[
P^{n+1}(+) = \left[ I - K^{n+1}H^{n+1} \right] P^{n+1}(-)
\]

**HINDCASTING ICE IN THE MISSOURI RIVER**

Oahe Dam is located on the Missouri River at Pierre, South Dakota, and forms Oahe Reservoir (Fig. 1). The flows in the Missouri River downstream of Oahe Dam are completely controlled by Oahe Dam, with the exception of the Bad River and other smaller tributaries. The overall movement of water from Oahe Dam is a function of the flow conditions of the reservoir system along the Missouri River. The Corps of Engineers’ Reservoir Control Center (RCC) in Omaha, Nebraska, schedules daily releases that must take into account the multipurpose uses of the reservoir system. Within this framework the hourly releases out of Oahe Dam are usually determined by the requirements for hydroelectric power production as determined by the Western Area Power Administration (WAPA). Consequently the flows in the Missouri River can change on an hourly basis. In fact, the flow out of Oahe Dam is rarely constant over a 24-hour period. It can change from a minimum of 0 cfs to a maximum of about 55,000 cfs. The presence of river ice decreases the hydraulic conveyance of the Missouri River and causes the stage in the river to rise. If the river ice cover has advanced into the Pierre area, the increased stages can potentially cause flooding at Pierre and Fort Pierre. The RCC reduces Oahe releases as river stages approach alert levels at any of the four gages downstream of Oahe Dam. The ability to forecast ice cover extent and river stages is potentially a valuable tool for the operators of the dam.

Simulation of the Missouri River when river ice is present is now addressed. The system error covariances were determined for the stages, discharges, and conveyance factors through numerical experiment that estimated the optimal values. The error covariances of the observations were set to reasonable values. However, whenever an water surface
observation fell outside of a physically appropriate range, which is indicative of a gage malfunction, the uncertainty associated with those measurements was set to an arbitrarily large value. As a result the state–space model would essentially ignore those measurements when updating the model.

Figure 1: The Missouri River downstream of Oahe Dam at Pierre, SD. Shown are the locations of river stage measurement gages and channel cross sections, which are identified by river mile.

Three periods in the recent past were selected for hindcasting by the state–space model. We will present only one period when ice was present in the Missouri River and observations are available: 19 December 1996 through 26 January 1997 This period was simulated without assimilation with varying success.

A primary problem that must be dealt with is the lack of observations of ice cover extent. Nineteen ice observations are available in 1996–97. This deficiency must be contrasted with approximately 940 hourly measurements of stages in the same period. If an ice observation was available, the observation was used to update the model. However, if no observation was available, as was most often the case, the state–space model ice cover extent was updated with the estimate of the ice cover extent that was produced by the ice cover progression model.

Useable observations of water temperature downstream of Oahe Dam are not available for the periods under consideration. Observations of ice in transport are also not available for these periods (or any periods of time, for that matter). As a result the thermal and ice transport state–space model is not used in these data assimilation experiments.

One basic question that must be addressed is the number of conveyance factors. Theoretically the number of conveyance factors could range from the number of cross sections (if a separate conveyance factor was used for each cross section) to one (if a single conveyance factor is applied at all cross sections). The approach used here was to follow a procedure analogous to the standard procedure for calibrating hydraulic model
roughness. In that procedure the number of gages along the channel that measure the water surface elevation controls the selection of conveyance factors. A separate conveyance factor can be used for each river reach that has a gage at its upstream and downstream end. In the case of the Missouri River there are five gages for which hourly measurements are available. (Fig. 1) The gages are, from upstream to downstream, the Oahe tailwater gage, the Pierre gage, the LaFramboise gage, the Farm Island Gage, and the Big Bend Dam gage. (The Big Bend Dam gage is the downstream boundary condition for the hydraulic model.) Thus, four conveyance factors can be used. The first would apply to the cross sections in the Oahe–Pierre reach; the second to the Pierre–LaFramboise reach; the third to the LaFramboise–Farm Island reach; and the fourth to the Farm Island–Big Bend reach.

SUMMARY: WINTER 1996–97
Figure 2 displays the estimated extent of the ice cover determined by the data assimilation experiment for the period 19 December 1996 through 26 January 1997. The updated results match the observations closely.

![Figure 2: Results of data assimilation experiment for 19 December 1996 through 26 January 1997. Shown is the location of the leading edge of the ice cover (solid line) and observations (crosses) in river miles (PIERRE = Pierre Gage; LFSD=La Framboise Island Gage; FISD = Farm Island gage).](image)

The updated stages essentially duplicate the observed stages except during periods when observations are missing or bad, such as 10–14 January for the Farm Island gage. The error covariance of the measurements of the Farm Island gage was set to a large value for the period 10–14 January so that measurements from this gage were ignored for this period.

There were other, subtler problems with the observations during this winter that dramatically affect the estimates of the conveyance factors shown in Figure 3. The water surface elevations measured at the Oahe tailwater gage, the Pierre gage, and the
LaFramboise gage nearly coincide during the period 10–14 January. It is not clear what is causing this result, perhaps a large ice blockage in the river. The only means the state–space model has for coping with these observations is to assume that the channel upstream of the LaFramboise gage has become much less rough so that the drop in stage between the gages is minimized. As a result the conveyance coefficients increased dramatically during this period for the reach between the Oahe tailwater gage and the Pierre gage and the reach between the Pierre gage and the LaFramboise gage.

Figure 3: Results of data assimilation experiment for 19 December 1996 through 26 January 1997. Shown are the conveyance factors estimated by the model for the Oahe–Pierre reach (OAHE-PIR); the Pierre–LaFramboise reach (PIR-LFSD); the LaFramboise–Farm Island reach (LFSD-FISD); and the Farm Island–Big Bend reach (FISD-BIG BEND)

The overall coefficient of determination for all the gages improved from .145 to .907 using data assimilation for this period. Data assimilation promises to improve performance of real-time operation of numerical river ice models.

REFERENCES: