MODELLING UNDER-ICE MOVEMENT OF COHESIVE SEDIMENTS: HAY RIVER, NORTHWEST TERRITORIES, CANADA

D. Milburn1 and B.G. Krishnappan2

ABSTRACT
An intensive field program was conducted just before river-ice breakup at the Hay River, Northwest Territories, Canada in April, 2000, followed by controlled laboratory experiments on Hay River water and sediments in a rotating circular flume at Burlington, Ontario, Canada to better understand the nature of cohesive sediment transport in the Hay River. Results from these earlier studies have shown that the deposition of fine sediment is possible in the shallower portion of the river along the river banks, where the bed shear stress is lower than the critical shear stress for deposition of the Hay River sediment during the winter months. The remobilization and the transverse dispersion of the sediment across the width of the river are attributed as possible causes for the formation of sediment plume just prior to breakup when the bed shear stress exceed the critical shear stress for the erosion. This hypothesis will be tested using a new modelling strategy proposed in this paper.

INTRODUCTION – THE NATURE OF COHESIVE SEDIMENTS
Cohesive sediments are characterized as a mixture of predominantly clay and silt-sized fractions of clay-type minerals but may also contain a range of organic compounds (Raudkivi, 1998). Much uncertainty exists about the hydraulic forces that erode, re-suspend and transport cohesive sediments, particularly the difference in the critical condition for initiation and cessation of sediment movement in a flowing medium. For coarse-grained material, erosion and deposition can occur simultaneously under a constant bed shear stress. For cohesive material, however, simultaneous erosion and deposition is not possible because different critical conditions occur - one for erosion and a different one for deposition.

Cohesive sediments tend to be transported not as single constituent grains, but rather in a flocculated/aggregated form (Droppo, 2001; Lick, 1982; Mehta and Partheniades, 1982).
1975; Partheniades and Kennedy, 1966). These “flocs” comprise a complex matrix of microbial communities, organic and inorganic particles and substantial interfloc spaces that allow retention or passage of water. Furthermore, because of the electrochemical forces in combination with biological factors, flocs will settle at rates completely different from those of their primary constituent particles (Lau and Krishnappan, 1992). As Droppo et al. (1997) explain these flocs are like individual microecosystems with autonomous and interactive physical, chemical and biological behaviours. Their reactivity and relatively large pore spaces form an important medium for removal of contaminants from a water column. Low flow conditions favour deposition of these flocs only to be resuspended during more dynamic conditions such as river-ice breakup resulting in an important vector for contaminant transport. Some of the main differences between the cohesionless and cohesive sediments are listed in Table 1.

Table 1: Comparison of Cohesionless and Cohesive Sediments

<table>
<thead>
<tr>
<th>Cohesionless Sediments</th>
<th>Cohesive Sediments</th>
</tr>
</thead>
<tbody>
<tr>
<td>• tend to move as single particles or grains</td>
<td>• form as agglomerates or “flocs”</td>
</tr>
<tr>
<td>• size ranges from silts (2 to 62 µm) to sands (&gt;62 µm)</td>
<td>• very fine clay (&lt;2 µm) and fine silt (&lt; 16 µm) particles</td>
</tr>
<tr>
<td>• erosion and deposition is a function of size and shear stress</td>
<td>• erosion and deposition a function of size, shear stress, mineralogy and chemical-biological properties</td>
</tr>
<tr>
<td>• can be eroded and deposited simultaneously</td>
<td>• cannot be eroded and deposited simultaneously</td>
</tr>
<tr>
<td>• can be transported as bed load</td>
<td>• rarely transported as bed load - remain in suspension</td>
</tr>
</tbody>
</table>

The transport relationships developed for cohesionless sediment are not adequate to predict the transport behaviour of cohesive sediment (Raudkivi, 1998; Shen and Julien, 1992). At the present state of knowledge, the fine sediment transport characteristics can only be obtained by direct measurements in special flumes such as rotating circular flumes (Krishnappan, 1993).

Previous work by Milburn and Prowse (2002) shows that the bed sediments deposited as winter accumulations at Hay River, Northwest Territories, Canada are cohesive in nature. It has also been shown (Milburn and Prowse, 1996; 1998) that the protracted period of winter-ice cover in northern rivers favours the accumulation of fine-grained (<62 microns), contaminant-bearing sediments. These sediment accumulations can be mobilized immediately prior to breakup when discharges and bed shear stress first start to rise from the winter low flow condition (Milburn and Prowse, 1996; 2002). The environmental implications of this phenomenon are largely unknown and poorly understood, but the spring breakup could represent a significant, episodic release of sediment-bound contaminants.
RESEARCH OBJECTIVES

In view of the above, the objective of this research was to extend the work of Milburn and Prowse (2002) and Milburn and Krishnappan (2002) by taking further detailed field measurements just before breakup that can be used for modelling cohesive sediment transport. Results from these earlier studies have shown that the deposition of fine sediment is possible along the shallower portion of the river along the river banks, where the bed shear stress is lower than the critical shear stress for deposition of the Hay River sediment during the winter months. The remobilization and the transverse dispersion of the sediment across the width of the river are attributed as possible causes for the formation of sediment plume just prior to breakup when the bed shear stress exceed the critical shear stress for the erosion. This hypothesis will be tested using a new modelling strategy proposed in this paper. The strategy consists of applying an advection-dispersion equation expressed in stream-tube coordinate system to model the sediment deposition in the river during winter months. The stream tubes will be constructed using measured flow depths and predicted velocity distributions in the lateral direction in a number of cross-sections. The water surface slopes that are required for predicting the velocity profiles will be obtained by applying the MOBED model, which solves the St. Venant equations and a sediment mass balance equation. With a better understanding of the cohesive forces governing erosion and deposition of these materials, a new modelling strategy to predict the transport of the sediment in the Hay River under ice-covered conditions is formulated. The details of the modelling strategy are described below.

A MODELLING STRATEGY TO PREDICT TRANSPORT OF FINE SEDIMENT

A fine sediment dispersion and deposition model developed by Krishnappan (1991) is adapted for modelling sediment transport in the Hay River. The model solves an advection-diffusion equation expressed in curvilinear coordinate system (see Figure 1). Some of the salient features of the model are discussed here for the sake of completeness. The governing equation solved in the model is shown below:

\[
\frac{\partial C}{\partial x} = \frac{\partial}{\partial \eta} \left( m_x U h^2 E_z \frac{\partial C}{\partial \eta} \right) + \frac{m_x \lambda_1}{U} C + \frac{m_z \lambda_2}{U h} \eta \tag{1}
\]

where \( C \) is the depth averaged volumetric concentration of the fine sediment, \( x \) is the distance measured along the longitudinal coordinate axis, \( U \) is the depth-averaged velocity component in the \( x \) direction, \( h \) is the flow depth, \( Q \) is the volumetric flow rate, \( E_z \) is the transverse dispersion coefficient, \( m_x \) is the metric coefficient of the coordinate system, \( \lambda_1 \) is the rate coefficient for reactions, \( \lambda_2 \) is the rate of source or sink and \( \eta \) is the normalized cumulative discharge defined as:

\[
\eta = \frac{1}{Q} \int_0^z m_z U h dz \tag{2}
\]

where \( z \) is the transverse distance coordinate and \( m_z \) is the metric coefficient in the \( z \) direction. Equation (1) expresses the mass balance of fine sediment as it undergoes advection and turbulent diffusion and dispersion in a steady flow in a natural stream when the sediment enters the reach as a steady source.
The boundary conditions that will be used for the application of the model in the Hay River are as follows:

(i) upstream boundary - the distribution of the sediment concentration at the first cross-section will be used as the upstream boundary condition. This condition can be obtained by direct measurements or from some other calculations;
(ii) side banks - the sediment flux across these boundaries is assumed to be zero; and
(iii) bed - the sediment deposition at the stream bed and the re-suspension of the deposited sediment are specified to the model in terms of the source-sink term $\lambda_2$.

The laboratory experiments described in Milburn and Krishnappan (2002) were used to determine the deposition and erosion fluxes for fine sediments from the Hay River. Only the deposition of the sediment is modelled during the winter months when the river is ice-covered and the flow rate is fairly steady. It is assumed that the re-suspension of all of the deposited sediment occurs during the break up period when the flow rate and the bed shear stress increase sharply. The deposition flux, therefore, is equated to $\lambda_2$ and can be expressed as:

$$\dot{\lambda}_2 = p \omega C$$  \hspace{1cm} (3)

where $p$ is a probability that a sediment floc reaching the sediment bed stays at the bed and $\omega$ is the settling velocity. Krone (1962) defined this probability in terms of the bed shear stress ($\tau$) and a critical shear stress ($\tau_{cd}$) for deposition that was defined as the shear stress above which none of the suspended sediment would deposit; that is,

$$p = \left(1 - \frac{\tau}{\tau_{cd}}\right)$$  \hspace{1cm} (4)
From the experiments of Milburn and Krishnappan (2002) this critical shear stress ($\tau_{cd}$) for Hay River sediments was estimated as 0.323 Pa. The settling velocity $\omega$ was also determined from these experiments. Values ranged between 0.025 and 0.08 mm s$^{-1}$ with a mean of 0.05 mm s$^{-1}$.

The development of the curvilinear coordinate system shown in Figure 1 requires the knowledge of the depth averaged velocity distribution in the lateral direction and the depth variation across the river. The depth variation across the river will be obtained by surveying the river at a number of cross sections and the depth averaged velocity distributions are computed using an approach proposed by Djordjevic (1993) and improved by Guan et al. (2002). According to this approach, the depth averaged momentum equation in the longitudinal direction is simplified as follows:

$$\frac{\partial (hU^2)}{\partial x} + \frac{\partial (hUV)}{\partial y} = -gh \frac{dz_w}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y} (h\tau_{xy}) - \frac{1}{\rho} \tau_{hx}$$  \hspace{1cm} (5)

In the equation above, $U$ and $V$ are the depth-averaged velocity components in the longitudinal and transverse directions respectively, $h$ is the water depth, $z_w$ is the water surface elevation above a datum, $g$ and $\rho$ are the gravitational acceleration and water density respectively, $\tau_{xy}$ is the depth average turbulent shear stress and $\tau_{hx}$ is the bed shear stress. The depth averaged turbulent shear stress is calculated using the eddy viscosity concept as:

$$\frac{\tau_{xy}}{\rho} = v_t \frac{\partial U}{\partial y}$$  \hspace{1cm} (6)

where $v_t$ is the depth-averaged eddy viscosity. Assuming similarity with mass transport, $v_t$ is expressed as

$$v_t = c_v h U,$$  \hspace{1cm} (7)

where $c_v$ is an empirical constant and $U_*$ is the shear velocity. The bed shear stress is expressed as:

$$\frac{\tau_{hx}}{\rho} = \frac{\tau_h}{\rho \cos \Phi} = \frac{U_*^2}{\cos \Phi} = \frac{c_f U^2}{\cos \Phi}$$  \hspace{1cm} (8)

where $\Phi$ is the transverse angle of inclination of the bed to the horizontal, $c_f$ is the friction factor, which can be expressed in terms of the Manning’s $n$ as:

$$c_f = gn^2 R^{-\frac{1}{3}} = gn^2 \left( \frac{h dy}{dy / \cos \Phi} \right)^{\frac{1}{3}} = gn^2 \left( h \cos \Phi \right)^{\frac{1}{3}}$$  \hspace{1cm} (9)

Substituting equations (6), (7) and (8) into equation (5) results in the following:
\[ gh \frac{dz_w}{dx} - \frac{c_v}{2} \frac{\partial}{\partial y} \left( h^2 \sqrt{c_f \frac{\partial U^2}{\partial y}} \right) + \frac{c_f U^2}{\cos \Phi} = - \frac{\partial \left( hU^2 \right)}{\partial x} - \frac{\partial (hUV)}{\partial y} \] (10)

Assuming that the first term on the right hand side of equation (10) is comparatively small and in the second term, the product UV can be expressed as follows:

\[ UV = C_{sw} U^2 \] (11)

generated by Guan et al. (2000), equation (10) can be simplified as follows:

\[ \frac{d}{dy} \left( \frac{c_v}{2} n h^{\frac{11}{6}} (\cos \Phi)^{\frac{1}{6}} g \frac{dU^2}{dy} \right) - \frac{d}{dy} \left( C_{sw} hU^2 \right) - gn^2 h^{\frac{1}{3}} (\cos \Phi)^{\frac{4}{3}} U^2 = gh \frac{dz_w}{dx} \] (12)

The above equation is a second order ordinary differential equation with variable coefficients and can be solved numerically once the water surface slope \((z_w)\) appearing on the right hand side of the equation is specified.

The determination of the water surface slope for equation (12) can be achieved by the use of MOBED — a one dimensional, mobile boundary, unsteady flow model (Krishnappan, 1981; 1983; 1986). The model solves the full St. Venant equations and a sediment mass balance equation to calculate the bed level changes in addition to water level changes as a function of time and distance along the river. The full details of the model can be found in the three references quoted above.

The MOBED model will be operated for the Hay River in two modes. In the first mode, MOBED will be run as a quasi-steady state model to predict the longitudinal variation of the water levels during the winter months, which will be used in equation (12) to calculate the velocity distribution in the lateral direction. In the second mode, the model will be run as an unsteady flow model to predict the increased flow velocities and the bed shear stresses and consequently the re-suspension potential of the Hay River flows during the breakup periods.

**Predicted lateral velocity distribution**

A sample velocity distribution predicted using equation (12) is shown in Figure 2. Measured velocities are also plotted as points in this figure. The velocity is normalized using the average velocity of 0.35 m s\(^{-1}\) determined from detailed under-ice velocity measurements and the lateral position in the river is normalized using the width of the river, which at this section is 65 m. It can be seen that the predicted velocity distribution approximates closely to the measured distribution. To predict this velocity distribution, a measured water surface slope at the study location was used. The presence of ice cover is accounted by using a combined Manning’s n for the ice cover and the bed roughness. The velocity distribution corresponds to the study location, just upstream of the Water Survey of Canada gauging site shown in Figure 3. The river reach that will be modelled spans from Paradise Gardens approximately three kilometers upstream and the Great Slave Lake approximately four kilometers downstream. A detailed surveying work to establish the shape of the cross sections in this reach was conducted during the summer of 2002.
SUMMARY AND CONCLUSIONS
A modelling strategy to predict the transport of fine sediment in the Hay River near the town of Hay River, Northwest Territories in Canada is proposed in this paper. The strategy targets the movement of sediment under ice cover during winter months and aims to explain the formation of sediment plume that is often observed just before the breakup of the ice cover in the spring. The modelling strategy consists of an advection-dispersion equation expressed in stream tube coordinate system, which required the knowledge of velocity and depth distribution in the reach of the river to be modelled. A
simple method to calculate the velocity distributions in the lateral direction knowing the depth distribution and the slope of the water surface is proposed for this modelling strategy. The slope of the water surface was evaluated using MOBED - an existing one dimensional unsteady flow model.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the technical support of Robert Stephens of the National Water Research Institute and of Shawne Kokelj of the Water Resources Division, Department of Indian Affairs and Northern Development in carrying out the detailed velocity distributions in the Hay River.

REFERENCES


Milburn D. and Prowse T.D. The effect of river-ice break-up on suspended sediment