ES250: Electrical Science

HW1: Electric Circuit Variables, Elements and Kirchhoff’s Laws
Introduction

• Engineers use electric circuits to solve problems that are important to modern society, such as:
  1. the generation, transmission, and consumption of electric power and energy
  2. the encoding, decoding, storage, retrieval, transmission, and processing of information

• In Chapter 1, we will do the following:
  1. Represent the current and voltage of an electric circuit element, paying attention to the reference current and voltage directions
  2. Use the passive convention to determine whether the power or energy is supplied or received by a circuit element
  3. Use scientific notation to represent electrical quantities within a wide range of magnitudes
Electric Circuits and Current

• An electric circuit (or network) is an interconnection of electric elements linked together in a closed path so that an electric current may flow continuously through it.

• Consider a simple circuit consisting of two well-known electrical elements: a battery and a resistor.

• In this case, each element (or device) is represented by a two-terminal (or node) element as shown.
Electric Circuits and Current

• Charge is the intrinsic property of matter responsible for electric phenomena:

1. The quantity of charge \( q \) can be expressed in terms of the charge of one electron which is \(-1.602 \times 10^{-19}\) coulombs

2. Charge may flow in an electric circuit and current is the time rate of change of charge past a given point, expressed as

\[
i = \frac{dq}{dt}
\]

3. The unit of current is the ampere (A); an ampere is 1 coulomb per second
Electric Circuits and Current

• A complete description of current requires both a value, which can be positive or negative, and a direction indicated by an arrow; thus we say that a current exists in or through an element.

![Diagram of current flow](image)

\[ i_1 = -i_2 \]

• Note, while the flow of current is historically represented as a flow of positive charges we realize that current flow in circuits is typically due to the flow of negatively charged electrons (this is not always the case in devices such as semiconductors).
Electric Circuits and Current

- If the current flowing through an element is constant, we represent it by the constant $I$, as shown below, and refer to it as a direct current (dc):

- A time-varying current $i(t)$ can take many forms, such as a ramp, a sinusoid called an alternating current (ac), or an exponential, as shown below:
Electric Circuits and Current

• If the charge $q$ is known, the current $i$ is readily found using

\[ i = \frac{dq}{dt} \]

• Example 1.2-1 Calculating Current from Charge: Find the current in an element when the charge entering the element is $q = 12t$ where $t$ is the time in seconds.

• Solution: Since the unit of charge is coulombs, C, the current is given by

\[ i = \frac{dq}{dt} = 12 \]

where the unit of current is amperes, A.
Electric Circuits and Current

- If the current $i$ is known, the charge $q$ is readily found using

\[ q = \int_{-\infty}^{t} i \, d\tau = \int_{0}^{t} i \, d\tau + q(0) \]

where $q(0)$ is the charge at $t = 0$

- Example 1.2-2 Calculating Charge from Current: Find the charge that has entered the terminal of an element from $t = 0$ s to $t = 3$ s when the current entering the element is as shown:
Solution

From Figure 1.2-6 we can describe \( i(t) \) as

\[
i(t) = \begin{cases} 
0 & t < 0 \\
1 & 0 < t \leq 1 \\
t & t > 1 
\end{cases}
\]

Using Eq. 1.2-2, we have

\[
q = \int_0^3 i(t) \, dt = \int_0^1 1 \, dt + \int_1^3 t \, dt
\]

\[
= t \bigg|_0^1 + \frac{t^2}{2} \bigg|_1^3 \\
= 1 + \frac{1}{2}(9 - 1) = 5 \quad \text{C}
\]

Alternatively, we note that integration of \( i(t) \) from \( t = 0 \) to \( t = 3 \) s simply requires the calculation of the area under the curve shown in Figure 1.2-6. Then, we have

\[
q = 1 + 2 \times 2 = 5 \quad \text{C}
\]
Systems of Units

- The fundamental or base units of SI are:

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>SI UNIT</th>
<th>NAME</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>meter</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>second</td>
<td>s</td>
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<tr>
<td>Electric current</td>
<td>ampere</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Thermodynamic temperature</td>
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<td>kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>mole</td>
<td>mole</td>
<td>mol</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>candela</td>
<td>candela</td>
<td>cd</td>
</tr>
<tr>
<td>QUANTITY</td>
<td>UNIT NAME</td>
<td>FORMULA SYMBOL</td>
<td>SYMBOL</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-------------------------------------------</td>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td>Acceleration — linear</td>
<td>meter per second per second</td>
<td>m/s²</td>
<td>N</td>
</tr>
<tr>
<td>Velocity — linear</td>
<td>meter per second</td>
<td>m/s</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>hertz</td>
<td>s⁻¹</td>
<td>Hz</td>
</tr>
<tr>
<td>Force</td>
<td>newton</td>
<td>kg · m/s²</td>
<td>N</td>
</tr>
<tr>
<td>Pressure or stress</td>
<td>pascal</td>
<td>N/m²</td>
<td>Pa</td>
</tr>
<tr>
<td>Density</td>
<td>kilogram per cubic meter</td>
<td>kg/m³</td>
<td></td>
</tr>
<tr>
<td>Energy or work</td>
<td>joule</td>
<td>N · m</td>
<td>J</td>
</tr>
<tr>
<td>Power</td>
<td>watt</td>
<td>J/s</td>
<td>W</td>
</tr>
<tr>
<td>Electric charge</td>
<td>coulomb</td>
<td>A · s</td>
<td>C</td>
</tr>
<tr>
<td>Electric potential</td>
<td>volt</td>
<td>W/A</td>
<td>V</td>
</tr>
<tr>
<td>Electric resistance</td>
<td>ohm</td>
<td>V/A</td>
<td>Ω</td>
</tr>
<tr>
<td>Electric conductance</td>
<td>siemens</td>
<td>A/V</td>
<td>S</td>
</tr>
<tr>
<td>Electric capacitance</td>
<td>farad</td>
<td>C/V</td>
<td>F</td>
</tr>
<tr>
<td>Magnetic flux</td>
<td>weber</td>
<td>V · s</td>
<td>Wb</td>
</tr>
<tr>
<td>Inductance</td>
<td>henry</td>
<td>Wb/A</td>
<td>H</td>
</tr>
</tbody>
</table>
SI Prefixes

- The great advantage of the SI system is that it incorporates a decimal system for relating larger or smaller quantities to the basic unit. An example of the common use of a prefix is the centimeter (cm), which is 0.01 meter. The standard prefixes given below:

<table>
<thead>
<tr>
<th>MULTIPLE</th>
<th>PREFIX</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
</tr>
<tr>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>femto</td>
<td>f</td>
</tr>
</tbody>
</table>
Voltage

- Voltage describes the energy required to cause charge to flow and we say that voltage exists across an element.
- The voltage notation (or references) consist of a value and an assigned direction given by its polarities (+, −).
- When considering $v_{ab}$, terminal a is called the “+ terminal” and terminal b is called the “− terminal”.

Note, voltages $v_{ab}$ and $v_{ba}$ have the same magnitude but opposite sign (polarity).

$v_{ab} = -v_{ba}$

$v_{ab}$ can be read as “the voltage at terminal a with respect to terminal b;” alternatively, we say that $v_{ab}$ is the voltage drop from terminal a to terminal b.
Voltage

• The unit of voltage is the volt, \( V \), and the equation for the voltage across the element is

\[
v = \frac{dw}{dq}
\]

where \( v \) is voltage, \( w \) is energy (or work), and \( q \) is charge

• Note, a charge of 1 coulomb delivers an energy of 1 joule as it moves through a voltage of 1 volt
Power and Energy

- **Power** is the time rate of supplying or absorbing energy:
  \[ p = \frac{dw}{dt} \]
  where \( p \) is power in watts, and \( w \) is energy in joules.

- The power associated with the current flow through an element is:
  \[ p = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \]

- The power and energy delivered to an element are of great importance. For example, the useful output of an electric lightbulb can be expressed in terms of power. We know that a 300-watt bulb delivers more light than a 100-watt bulb.
The Passive Convention

**POWER ABSORBED BY AN ELEMENT**

Since the reference directions of $v$ and $i$ adhere to the passive convention, the power

$$p = vi$$

is the power absorbed by the element.

---

**POWER SUPPLIED BY AN ELEMENT**

Since the reference directions of $v$ and $i$ do not adhere to the passive convention, the power

$$p = vi$$

is the power supplied by the element.
Power and Energy

• When the element voltage and current adhere to the passive convention, the energy absorbed by an element can be determined from:

\[ w = \int_{-\infty}^{t} p \, d\tau = \int_{0}^{t} p \, d\tau \]

• Example 1.5-1 Electrical Power and Energy: Consider the element shown labeled using the passive convention with \( v = 4 \text{ V} \) and \( i = 10 \text{ A} \). Find the power absorbed by the element and the energy absorbed over a 10-s interval

**Solution**

The power absorbed by the element is

\[ p = vi = 4 \cdot 10 = 40 \text{ W} \]

The energy absorbed by the element is

\[ w = \int_{0}^{10} p \, dt = \int_{0}^{10} 40 \, dt = 40 \cdot 10 = 400 \text{ J} \]
Example 1.5-2 Electrical Power and the Passive Convention

Consider the element shown in Figure 1.5-2. The current $i$ and voltage $v_{ab}$ adhere to the passive convention, so the power absorbed by this element is

$$\text{power absorbed} = i \cdot v_{ab} = 2 \cdot (-4) = -8 \text{ W}$$

The current $i$ and voltage $v_{ba}$ do not adhere to the passive convention, so the power supplied by this element is

$$\text{power supplied} = i \cdot v_{ba} = 2 \cdot (4) = 8 \text{ W}$$

As expected

$$\text{power absorbed} = - \text{ power supplied}$$
Resistors

• Resistance is the physical property of an element or device that impedes the flow of current; it is represented by the symbol $R$

• For the passive convention, Ohm's law relates voltage and current: $v = Ri$

• Schematic symbol for a resistor with resistance of $R$ ohms

• What is the expression for Ohm’s law that adheres to the passive convention? $v = Ri_a$
Resistors

• The power delivered to a resistor (when the passive convention is used) is:

\[ p = vi = v \left( \frac{v}{R} \right) = \frac{v^2}{R} \]

• Alternatively, since \( v = iR \), we can write:

\[ p = vi = (iR)i = i^2R \]

• The energy delivered to a resistor is:

\[ w = \int_{-\infty}^{t} pd\tau = \int_{-\infty}^{t} i^2Rd\tau \]

• Since the energy absorbed is always nonnegative, the resistor is referred to as a passive element
Example 2.4-1 Power Dissipated by a Resistor

Solution

According to Ohm's law, Eq. 2.4-4, we have

\[ v = Ri \]

Since \( v = 12 \text{ V} \) and \( R = 6 \Omega \), we have \( i = 2 \text{ A} \).

In order to find the power delivered by the battery, we use

\[ p = vi = 12(2) = 24 \text{ W} \]

Finally, the energy delivered in the four-hour period is

\[ w = \int_0^t pd\tau = 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 \text{ J} \]
Independent Sources

• Sources are intended to supply energy to a circuit; there are two types: voltage sources and current sources:

  Voltage source:
  \[ v(t) = 12 \cos 1000t \text{ or } v(t) = 9 \text{ or } v(t) = 12 - 2t \]

  Current source:

• The voltage \( v(t) \) of a voltage source is specified but its current is determined by the rest of the circuit; while the converse is true for a current source.
Short and Open Circuits

- A short circuit is an ideal voltage source having $v(t) = 0$ with the current determined by the rest of the circuit, while an open circuit is an ideal current source having $i(t) = 0$ with the voltage across an open circuit is determined by the rest of the circuit.

- Note, the power absorbed by these devices is zero.
Voltmeters and Ammeters

• A meter is connected to a circuit using terminals called probes to measure voltage or current

• Probes are color coded to indicate the reference direction of the variable being measured, e.g., red for the positive terminal and black for the negative terminal.
Voltmeters and Ammeters

- Note, the voltmeter is connected in parallel with the voltage to be measured using the passive convention. The current flow into an ideal voltmeter is zero, meaning that the ideal voltmeter dissipates zero power.
Voltmeters and Ammeters

- Note, the ammeter is connected in series with the current to be measured using the passive convention. The voltage across an ideal ammeter is zero, meaning that the ideal ammeter dissipates zero power.
Voltmeter and Ammeter Examples

• Circuit measurements:

• Equivalent circuit:
Voltmeter and Ammeter Examples
Voltmeter and Ammeter Examples
Switches

- Switches have two distinct states: open and closed
  - Ideally, a switch acts as a short circuit when it is closed and as an open circuit when it is open
- We’ll consider two types of switches: single-pole, single-throw (SPST) and single-pole, double-throw (SPDT):
Switches

- Switches have two distinct states: open and closed
  - Ideally, a switch acts as a short circuit when it is closed and as an open circuit when it is open
- We’ll consider two types of switches: single-pole, single-throw (SPST) and single-pole, double-throw (SPDT):

![Diagram of switch states and operations]
Example 2.9-1: Switches

- Based on the schematic in red, when do the others take place? (If needed, see text for solution)
Exercise 2.9-2: Make-before-break SPDT Switch

- What is the value of the voltage $v$ at time $t = 4$ s? At $t = 6$ s?

Solution:

\[
\nu(t) = \begin{cases} 
IR = (2\text{m})(3k) = 6\text{V}, & \text{for } t < 5\text{s} \\
0\text{V}, & \text{for } t > 5\text{s}
\end{cases}
\]
Kirchhoff's Laws

• An electric circuit consists of circuit elements connected together by nodes
  – It is common practice to draw circuits using straight lines positioning the elements horizontally or vertically

These equivalent circuits have six elements (1-6) and fours nodes (a-d)
Example 3.2-1: Different Drawings of the Same Circuit

• The same circuit can be drawn in several different looking ways, but drawings represent the same circuit if corresponding elements are connected to corresponding nodes.

Are these two circuits equivalent? If so, why?
Kirchhoff’s Current Law (KCL)

• Kirchhoff’s current law states that the algebraic sum of the currents entering any node is identically zero for all instants of time

• The phrase *algebraic sum* indicates that we must take reference directions into account as we add up the currents of elements connected to a particular node
  
  – one way to take reference directions into account is to use a plus sign when the current is directed away from a node and a minus sign when the current is directed toward a node, or currents in equal to currents out
Kirchhoff’s Current Law (KCL)
• Write the KCL equation for node a:

\[-i_1 + i_2 + i_3 - i_4 = 0 \implies i_2 + i_3 = i_1 + i_4\]
Kirchhoff’s Voltage Law (KVL)

- Kirchhoff’s voltage law states that the algebraic sum of the voltages around any loop in a circuit is identically zero for all time.

- A *loop* is a closed path through a circuit that does not encounter any intermediate node more than once.

How many loops does this circuit contain?
Kirchhoff’s Voltage Law (KVL)

- The phrase *algebraic sum* indicates that we must take polarity into account as we add up the voltages of elements that comprise a loop.
  - One way to do this is to write the voltage with a plus sign when we encounter the + of the polarity before the − and a minus sign when we encounter the − of the polarity before the +.

Write the KVL eqn. associated with the loop containing elements 3-6 of the circuit:

\[ v_4 - v_5 - v_6 - v_3 = 0 \]
Example 3.2-3: Ohm's and Kirchhoff's Laws

• Consider the circuit below noting the passive convention was used to assign references and find all currents and voltages when $R_1 = 8 \, \Omega$, $v_2 = -10 \, V$, $i_3 = 2 \, A$, and $R_3 = 1 \, \Omega$. Also, determine the resistance $R_2$. 

[Image of the circuit diagram]
Solution

The sum of the currents entering node a is
\[ i_1 - i_2 - i_3 = 0 \]

Using Ohm’s law for \( R_3 \), we find that
\[ v_3 = R_3 i_3 = 1(2) = 2 \text{ V} \]

Kirchhoff’s voltage law for the bottom loop incorporating \( v_1 \), \( v_3 \), and the 10-V source is
Therefore,
\[ -10 + v_1 + v_3 = 0 \]
\[ v_1 = 10 - v_3 = 8 \text{ V} \]

Ohm’s law for the resistor \( R_1 \) is
\[ v_1 = R_1 i_1 \]

or
\[ i_1 = v_1 / R_1 = 8 / 8 = 1 \text{ A} \]

Since we have now found \( i_1 = 1 \text{ A} \) and \( i_3 = 2 \text{ A} \)
\[ i_2 = i_1 - i_3 = 1 - 2 = -1 \text{ A} \]

We can now find the resistance \( R_2 \) from
\[ v_2 = R_2 i_2 \]

or
\[ R_2 = v_2 / i_2 = -10 / -1 = 10 \text{ \Omega} \]
Example 3.2-4 Ohm's and Kirchhoff's Laws

• Determine the value of the current, in amps, measured by the ammeter:

Equivalent circuit, replacing the ammeter with a short circuit with an appropriate reference direction:
Solution

Applying KCL at node b gives

\[-i_a - 3i_a - i_m = 0\]

Applying KVL to closed path a-b-c-e-d-a

\[0 = -4i_a + 2i_m - 12 = -4\left(-\frac{1}{4}i_m\right) + 2i_m - 12 = 3i_m - 12\]

Finally, solving this equation gives

\[i_m = 4\text{ A}\]
Dependent Sources

• Dependent sources model the situation in which the voltage or current of one circuit element is proportional to the voltage or current of the second circuit element
  – used to model electronic devices such as transistors or amplifiers, e.g., the output voltage of an amplifier is proportional to the voltage input of that amplifier

• Each dependent source consists of two parts: the controlling part and the controlled part
  – the controlling part is either an open circuit or a short circuit
  – the controlled part is either a voltage source or a current source

• There are four types of dependent source that correspond to the four ways of choosing a controlling part and a controlled part, called:
  – voltage-controlled voltage sources (VCVS), current-controlled voltage sources (CCVS), voltage-controlled current sources (VCCS), and current-controlled current sources (CCCS)
## Types of Dependent Sources

<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current-Controlled Voltage Source (CCVS)</td>
<td><img src="image" alt="CCVS Symbol" /></td>
</tr>
<tr>
<td>$r$ is the gain of the CCVS.</td>
<td>$v_c = 0$</td>
</tr>
<tr>
<td>$r$ has units of volts/ampere.</td>
<td>$i_c$</td>
</tr>
<tr>
<td></td>
<td>$v_d = ri_c$</td>
</tr>
<tr>
<td>Voltage-Controlled Voltage Source (VCVS)</td>
<td><img src="image" alt="VCVS Symbol" /></td>
</tr>
<tr>
<td>$b$ is the gain of the VCVS.</td>
<td>$i_c = 0$</td>
</tr>
<tr>
<td>$b$ has units of volts/volt.</td>
<td>$v_c$</td>
</tr>
<tr>
<td></td>
<td>$v_d = bv_c$</td>
</tr>
</tbody>
</table>
Types of Dependent Sources

Voltage-Controlled Current Source (VCCS)

$g$ is the gain of the VCCS.

$g$ has units of amperes/volt.

Current-Controlled Current Source (CCCS)

$d$ is the gain of the CCCS.

$d$ has units of amperes/ampere.
Transistor Example

[Diagrams of transistor and equivalent circuits]
Exercise 2.7-1

• Find the power absorbed by the CCCS:

Since the CCCS references use the passive convention, the power absorbed is given by:

\[ p = i_d v_d = (4i_c)v_d = (4 \cdot -1.20) \cdot 24.0 = -115.2 \text{ W} \]

This implies that the CCCS is actually supplying energy to the circuit!
Transducers

- Transducers are devices that convert physical quantities to electrical quantities:
  - potentiometers convert position to resistance
  - temperature sensors convert temperature to current

The symbol \( R_p \) and \( (1 - a)R_p \) and \( aR_p \) A model for the potentiometer
Example 2.8-1 Potentiometer Circuit

\[
\nu_m = R_p I a = \frac{R_p I}{360} \theta \quad \Rightarrow \quad \theta = \frac{360}{R_p I} \nu_m
\]

- Suppose \( R_p = 10 \, \text{k}\Omega \) and \( I = 1 \, \text{mA} \), an angle of 163° would cause an output of \( \nu_m = 4.53 \, \text{V} \). Alternately, a meter reading of 7.83 V would indicate that \( \Theta = 282° \).
Questions?