EE/ME/AE324: Dynamical Systems

Chapters 10-11: Electromechanical, Thermal and Fluid Systems
Electromechanical Coupling by Magnetic Fields

- Many electromechanical devices contain current-carrying wires that can move within a magnetic field, e.g., motors, speakers, microphones, solenoids, transducers, etc.
- These devices are based on two phenomena that result from the Lorentz force law associated with Maxwell's eqns.:
  1. A current carrying wire in a magnetic field will experience a force on it, e.g., $d\mathbf{f}_e = i (d\mathbf{l} \times \mathbf{B})$
     \[ \Rightarrow f_e = B \ell i \] (right-hand rule applies)
  2. A voltage will be induced in a wire that moves relative to a magnetic field, e.g., $de_m = (\mathbf{v} \times \mathbf{B}) d\ell$
     \[ \Rightarrow e_m = B \ell v \] (right-hand rule applies)
Electromechanical Coupling by Magnetic Fields

• The variables needed to model such devices are:
  \( f_e \), the force on the conductor [N]
  \( v \), the velocity of the conductor with respect to the magnetic field [m/s]
  \( l \), the length of the conductor in the magnetic field [m]
  \( \phi \), the flux [Wb (webers)]
  \( f_e \), the flux density of the magnetic field [Wb/m\(^2\) or T(tesla)]
  \( i \), the current in the conductor [A]
  \( e_m \), the voltage induced in the conductor [V]
Electromechanical Coupling by Magnetic Fields

- The force on a conductor can be visualized using the RHR:

\[ f_e = Bli \]
Electromechanical Coupling by Magnetic Field

- The voltage induced on a conductor can also be visualized using the RHR:

\[ e_m = B\ell v \]
Electromechanical Coupling by Magnetic Field

- Representation of power flow for electromechanical systems:
  Power flow from electrical-to-mechanical (left), e.g., a motor, and mechanical-to-electrical (right), e.g., a generator.
Example System: The Galvanometer (Ammeter)

- A galvanometer is a type of ammeter: an instrument for detecting and measuring electric current.
- It's an analog transducer that produces a rotary deflection in response to an electric current flowing through its coil.

Assume the magnetic field is supplied by a permanent magnet with uniform flux density \( B \) directed from its N to S pole and its coil consists of \( N \) rectangular turns, each of radius \( a \) and length \( \ell \).
Example System: The Galvanometer (Ammeter)

- The electro-mechanical model for the galvanometer includes electrical and mechanical subsystem dynamics connected by an algebraic torque coupling, as shown below:

\[ J\ddot{\theta} + B\dot{\theta} + K\theta = \tau_e(i), \]

where \( \tau_e(i) = (2NB\ell a)i = \alpha i \)

where \( e_m(\dot{\theta}) = (2NB\ell a)\dot{\theta} = \alpha \dot{\theta} \)
Connection to Your Semester Project

- The motor assumed in your project functions similar to a galvanometer except that it is a linear rather than rotary device.

\[ v_i, f_{ei} = \alpha l_i \]

Note: For purposes of the model development, the power flow was assumed to be from the electrical to the mechanical subsystems, but this can be reversed by the controller when convenient, i.e., to transform energy from road-induced motion back into the car battery.

Road induced compression of the actuator can cause current to flow in the opposite direction, e.g., supplying energy to \( e_i(t) \).
The Mechanical Subsystem

- Assume all components stretched, resulting in the component side forces shown below:
Modeling Thermal Systems

- Thermal systems involve the storage and flow of heat energy, e.g., building HVAC systems, and are based on the laws of thermodynamics, e.g., ES340

- Generally, thermal systems are distributed and thus modeled using partial differential equations; we shall consider lumped parameter models which approximate the system and result in ordinary diff. eqns. with which we are familiar

- Systems that involve phase changes, such as boiling or condensation, are beyond the scope of this class as well as the steady-state analysis of thermodynamic cycles that may be required in the design of a chemical process
The Variables

- The variables used to describe thermal systems are:
  \( \theta \), temperature [K (kelvins)] \( \sim \) pressure variable, e.g., voltage
  \( q \), heat flow rate [J/s or W] \( \sim \) flow variable, e.g., current

- Generally, thermal systems are characterized by their variation about an operating point, e.g., using the incremental variables:
  \[ \hat{\theta}(t) = \theta(t) - \bar{\theta} \quad \text{and} \quad \hat{q}(t) = q(t) - \bar{q} \]

- The ambient temperature of the environment is denoted \( \theta_a \), and when \( \bar{\theta} = \theta_a \), \( \hat{\theta} \) is called a relative temperature
Element Laws

- Thermal capacitance $C$ [J/K] relates the rate of temperature change to the instantaneous net heat flow rate into a body:

$$\dot{\theta}(t) = \frac{1}{C} \left[ q_{in}(t) - q_{out}(t) \right]$$

$$\Rightarrow \theta(t) = \theta(t_0) + \frac{1}{C} \int_{t_0}^{t} \left[ q_{in}(\lambda) - q_{out}(\lambda) \right] d\lambda$$

- From an impedance perspective, thermal capacitance is:

$$Z_C(s) = \frac{\theta(s)}{q_{in}(t) - q_{out}(t)} = \frac{1}{Cs}$$

- This is analogous to an electrical capacitance; thus thermal systems can be modeled using an equivalent circuit!
Element Laws

• Thermal resistance $R$ [K/W] due to the flow of heat by conduction from one body to another through a medium connecting them:

$$q(t) = \frac{1}{R} [\theta_1(t) - \theta_2(t)]$$

• From an impedance perspective, thermal resistance is defined:

$$Z_R(s) = \frac{[\theta_1(s) - \theta_2(s)]}{q(s)} = R$$

• This is analogous to an electrical resistance; thus thermal systems can be modeled using an equivalent circuit!
Thermal Sources

• Two types of ideal thermal sources are possible:
  1. A source that adds/removes heat at a specified rate $q_i(t)$

  ![Diagram of heat flow](image)

  $q_i(t)$

  2. A source that directly specifies the temperature of a body $\theta_i(t)$ regardless of the rate at which heat flows between the body and the rest of the system
Ex11.3: One C Thermal System

- The vessel below has a thermal capacitance C enclosed by insulation having a thermal resistance R; the temperature within the vessel is assumed uniform at $\theta$, while the ambient temperature is assumed to be at $\theta_a$; heat is being added to the vessel at a rate $q_i(t)$
Ex11.3: One C Thermal System

- The heat flow into and out of the vessel are given by:

\[ q_{in}(t) = q_i(t) \quad \text{and} \quad q_{out}(t) = \frac{1}{R} \left( \theta - \theta_a \right) \]

Substituting these values into the element law for thermal capacitance yields:

\[
\dot{\theta} = \frac{1}{C} \left[ q_{in}(t) - q_{out}(t) \right] = \frac{1}{C} \left[ q_i(t) - \frac{1}{R} \left( \theta - \theta_a \right) \right]
\]

\[
\Rightarrow \dot{\theta} + \frac{1}{RC} \theta = \frac{1}{C} q_i(t) + \frac{1}{RC} \theta_a
\]
Ex11.3: One C Thermal System

- At the OP the model reduces to $\bar{\theta} = Rq_i + \bar{\theta}_a$, so that the incremental model becomes $\dot{\theta} + \frac{1}{RC} \dot{\theta} = \frac{1}{C} q_i(t)$; this can be expressed in terms of the transfer function

$$H(s) = \frac{\hat{\theta}(s)}{\hat{q}_i(s)} = \frac{1}{s + \frac{1}{RC}},$$

whose step response is given by $\hat{\theta}(t) = R(1 - e^{-t/RC})$

$\Rightarrow \theta(t) = \bar{\theta} + \hat{\theta}(t) = Rq_i + \bar{\theta}_a + R(1 - e^{-t/RC})$
Ex11.3: One C Thermal System

- An equivalent electrical circuit can be created by representing thermal resistances using resistors, thermal capacitances using capacitors, heat flows replaced by electrical currents and temperatures replaced by voltages, as shown:

\[ q_{out} = \frac{1}{R} (\theta - \theta_a) \]

\[ \dot{\theta} = \frac{1}{C} \left[ q_i - \frac{1}{R} (\theta - \theta_a) \right] \]
Modeling a Batch Process

- Due to perfect mixing, assume an insulated vessel is filled with a liquid kept at uniform temperature $\theta_L$, the temp. of the heater is $\theta_H$ and the ambient temp. is $\theta_a$; the thermal resistance of the vessel to heater and vessel to ambient is $R_{HL}$ and $R_{La}$, respectively; the thermal capacitance of the heater and vessel are $C_H$ and $C_L$, respectively; the rate which heat is added to the heating element is $q_i(t)$, and the rates at which heat is transferred to the liquid and ambient air is $q_{HL}$ and $q_{La}$, respectively.
Modeling a Batch Process

- We can write the state-variable model as:

\[
\dot{\theta}_H = \frac{1}{C_H} [q_i(t) - q_{HL}] \quad \text{and} \quad \dot{\theta}_L = \frac{1}{C_L} [q_{HL} - q_{La}]
\]

where \( q_{HL} = (\theta_H - \theta_L) / R_{HL} \) and \( q_{La} = (\theta_L - \theta_a) / R_{La} \)

\[
\Rightarrow \dot{\theta}_H = - \left( \frac{1}{R_{HL} C_H} \right) \theta_H + \left( \frac{1}{R_{HL} C_H} \right) \theta_L + \left( \frac{1}{C_H} \right) q_i(t)
\]

\[
\dot{\theta}_L = \left( \frac{1}{R_{HL} C_L} \right) \theta_H - \left( \frac{1}{R_{HL} C_L} + \frac{1}{R_{La} C_L} \right) \theta_L + \left( \frac{1}{R_{La} C_L} \right) \theta_a
\]

This system has two inputs, e.g., \( q_i(t) \) and \( \theta_a \), that map into two outputs \( \theta_H \) and \( \theta_L \).
Modeling a Batch Process

- Equivalent electrical circuit model:
Modeling a Continuous Process

- Assume an insulated vessel is filled with a liquid kept at uniform temperature $\theta$ via perfect mixing, liquid enters at a constant volumetric flow rate of $\bar{w}$ [m$^3$/s] and temp. $\theta_i(t)$ and exits at the same flow rate at a temp. of $\theta$; the thermal resistance of the vessel is $R$ and the ambient temp. is a constant $\theta_a$; heat is added to the vessel at a rate of $q_h(t)$, the volume of the vessel is $V$, the liquid has a denisty of $\rho$ [kg/m$^3$] and a specific heat of $\sigma$ [J/kg-K]
Modeling a Continuous Process

- From the thermal capacitance law:

\[ \dot{\theta} = \frac{1}{C} \left[ q_{in} - q_{out} \right] \]

where \( q_{in} = q_h(t) + \bar{w} \rho \sigma \theta_i(t) \), \( q_{out} = \bar{w} \rho \sigma \theta + \frac{1}{R} \left( \theta - \theta_a \right) \)

and \( C = \rho \sigma V \); combining these expressions yields:

\[
\dot{\theta} + \left( \frac{\bar{w}}{V} + \frac{1}{RC} \right) \theta = \frac{1}{C} q_h(t) + \frac{\bar{w}}{V} \theta_i(t) + \frac{1}{RC} \theta_a
\]
Modeling a Continuous Process

• Expressing this in terms of the incremental variables yields:

\[
\dot{\hat{\theta}} + \frac{1}{\tau} \hat{\theta} = \frac{1}{C} \dot{q}_h(t) + \frac{\bar{w}}{V} \hat{\theta}_i(t)
\]

This system has two inputs, e.g., \(\dot{q}_h(t)\) and \(\hat{\theta}_i(t)\), that map into one output \(\hat{\theta}(t)\); since the system is linear, the solution can be found by considering each input independently, e.g.,

\[
\hat{\theta}(s) = H_1(s)\bigg|_{\dot{\theta}(s) = 0} \dot{q}_h(s) + H_2(s)\bigg|_{\dot{q}_h(s) = 0} \hat{\theta}_i(s)
\]

\[
= \begin{bmatrix}
\frac{1}{C} \\
\frac{\bar{w}}{V} \\
\frac{1}{s + \frac{1}{\tau}} \\
\frac{1}{s + \frac{1}{\tau}}
\end{bmatrix} \dot{q}_h(s) + \begin{bmatrix}
\frac{\bar{w}}{V} \\
\frac{1}{s + \frac{1}{\tau}}
\end{bmatrix} \hat{\theta}_i(s)
\]
Questions?