All problems are to be done according to the AISC Load and Resistance Factor Design. Values of yield stress $F_Y$ and tensile strength $F_u$ are available in AISC p. 2-39. Be sure to consider lateral-torsional buckling (LTB) as lateral support of the compression flange is only provided at the indicated locations.

9.1 For cases (a) and (b) determine the maximum concentrated service load $P$ that can act at midspan on a simply supported beam. Lateral supports exist only at the ends of the span. The service load is 65% live load and 35% dead load. Consider the beam weight in addition to the service load when calculating $M_u$. However, beam weight may be neglected for evaluating $C_b$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Section</th>
<th>Span (ft)</th>
<th>Steel Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>W30x99</td>
<td>30</td>
<td>A992 ($F_Y=50$ ksi)</td>
</tr>
<tr>
<td>b</td>
<td>W21x62</td>
<td>20</td>
<td>A36</td>
</tr>
</tbody>
</table>

9.2 For cases (a) and (b) select the lightest W section as a beam. Assume only flexure must be considered (i.e. neglect shear and deflection). The dead load given is in addition to the weight of the beam. Note the following conservative simplifying assumption may be made: If within an unbraced segment $L_b$, the difference between the maximum and the minimum bending moments is less than five percent, the moment diagram for that $L_b$ segment may be assumed to be uniform and therefore $C_b=1$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$W_D$ Dead Load (kip/ft)</th>
<th>$W_L$ Live Load (kip/ft)</th>
<th>Span Length L (ft)</th>
<th>Steel Grade</th>
<th>Lateral Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
<td>1.0</td>
<td>28</td>
<td>A572 Grade 60</td>
<td>Ends and midspan</td>
</tr>
<tr>
<td>b</td>
<td>0.2</td>
<td>0.6</td>
<td>35</td>
<td>A572 Grade 65</td>
<td>Every 7 feet</td>
</tr>
</tbody>
</table>
Problem 9.18. Determine the maximum concentrated load $P$ that can act at midspan on a simply supported span of 30 ft. Lateral supports exist only at the ends of the span. The service load is 65% live load and 35% dead load. The section is W30×99 of $F_y = 50$ ksi steel.

**Use LRFD Specification.**

(a) Obtain factored conc load $W_u$:

$$W_u = 1.2(0.35W) + 1.6(0.65W) = 1.46W$$

- $W$ - \[\text{30'-0}\]

(b) Determine the $C_b$ factor [LRFD-F1.2a].

$$C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

- $M_{\text{max}}$ = max moment in the unbraced segment = $M$
- $M_A$ = moment at 1/4 pt of unbraced segment = 0.5$M$
- $M_B$ = moment at midpoint of unbraced segment = $M$
- $M_C$ = moment at 3/4 pt of unbraced segment = 0.5$M$

$$C_b = \frac{12.5M}{2.5M + 3(0.5M) + 4M + 3(0.5M)} = 1.32 \text{ (also on p. 3-10)}$$

(c) Compute the design moment strength. From the LRFD Manual properties, p. 3-15, for W30×99 of $F_y = 50$ ksi steel:

$$\phi_bM_p = 1170 \text{ ft-kips}; \quad \phi_bM_r = 706 \text{ ft-kips}; \quad L_p = 7.42\text{ft}; \quad L_r = 21.4 \text{ ft}$$

Since $[L_b = 30 \text{ ft}] > [L_r = 21.4]$, elastic lateral-torsional buckling applies: (F2-3)

$$F_{cr} = \frac{C_b \pi^2 E}{(\frac{L_b}{C_t})^2} \sqrt{1 + 0.078 \frac{J_c}{S_x} \frac{(L_b)}{C_t}} \cdot \frac{I_x}{S_x} = \frac{\sqrt{128(26800)}}{269} = 6.885$$

$$C_t = 6.885 = 2.624''; \quad J = 3.77 \text{ in}^4; \quad c = 1; \quad h_o = 29''$$

$$F_{cr} = \frac{132 \pi^2 29800}{(\frac{30 \times 12}{2.624})^2} \sqrt{1 + 0.078 \frac{3.77}{269}(\frac{30 \times 12}{2.624})} = 26.244 \text{ ksi}$$

$$\phi M_n = 0.9(26.244) 269 = 6354 \text{ k-in} = 529.5 \text{ k-ft}$$

*No need to check local buckling of flange or web. For grade 50 noncompact would be indicated p. 3-15*

$$M_u = \frac{W_u L}{4} + \frac{w_u (\text{bm wt}) L^2}{8} = \frac{1.46W(30)}{4} + \frac{1.2(0.099)(30)^2}{8} = 10.95W + 13.37$$

529.5 = 10.95W + 13.37; $W = 47.1$ kips Maximum Service Load
Problem 9.1b. Determine the maximum concentrated load $P$ that can act at midspan on a simply supported span of 20 ft. Lateral supports exist only at the ends of the span. The service load is 65% live load and 35% dead load. The section is W21x62 of A36 steel.

Use LRFD Specification.

(a) Obtain factored conc load $W_u$:
$$W_u = 1.2(0.35W) + 1.6(0.65W) = 1.46W$$

(b) Determine the $C_b$ factor [AISC F1-1].
$$C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

- $M_{\text{max}} = \text{max moment in the unbraced segment} = M$
- $M_A = \text{moment at 1/4 pt of unbraced segment} = 0.5M$
- $M_B = \text{moment at midpoint of unbraced segment} = M$
- $M_C = \text{moment at 3/4 pt of unbraced segment} = 0.5M$

(Also see p. 3-10)
$$C_b = \frac{12.5M}{2.5M + 3(0.5M) + 4M + 3(0.5M)} = 1.32$$

(c) Compute the design moment strength. The computed properties, for $F_y = 36 \text{ ksi}$ steel are:
$$\phi M_r = 0.9(0.75x F_y)$$

- $\phi_b M_p = 389 \text{ ft-kips}$; $\phi_b M_r = 240 \text{ ft-kips}$; $L_p = 7.4 \text{ ft}$; $L_r = 22.26 \text{ ft}$

Since $L_p < [L_b = 20 \text{ ft}] < L_r$, the linear inelastic transition applies:

$$\phi_b M_n = C_b \left[ \phi_b M_p - (\phi_b M_p - \phi_b M_r) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right]$$
$$= 1.32 \left[ 389 - (389 - 240) \left( \frac{20 - 7.4}{22.26 - 7.4} \right) \right] = 1.32(263) = 347 \text{ ft-kips}$$

Thus, $\phi_b M_n = 347 \text{ ft-kips}$ because it does not exceed $\phi_b M_p = 389 \text{ ft-kips}$

The flange and web local buckling limit states do not control because

$$\left[ \frac{b_f}{2t_f} = 6.7 \right] < \left[ \lambda_p = \frac{0.38\sqrt{F_y}}{F_y} = 10.8 \right]; \quad \left[ \frac{h}{t_w} = 46.9 \right] < \left[ \lambda_p = \frac{3.76\sqrt{F_y}}{F_y} = 107 \right]$$

$$M_u = \frac{W_u L}{4} + \frac{w_u (\text{bm wt}) L^2}{8} = \frac{1.46W(20)}{4} + \frac{1.2(0.062)(20)^2}{8} = 7.3W + 3.72$$

$347 = 7.3W + 3.72; \; W = 47.0 \text{ kips}$ Maximum Service Load
Problem 9.25. Select the lightest W section as a simply supported beam on a 28 ft span. The load is 0.5 kip/ft dead load plus the beam weight, and the live load is 1.0 kips/ft. Lateral supports exist at the ends and midspan. Assume only flexure must be considered; i.e., omit treating shear and deflection. \( F_y = 60 \text{ ksi} \).

Use LRFD Specification.
(a) Obtain factored moment \( M_u \):
\[
w_u = 1.2(0.5 + \text{beam}) + 1.6(1.0) = 2.20 \text{ kips/ft}
\]
\[
M_u = \frac{w_u}{2}(28)^2 = 216 \text{ ft-kips (w/o beam)}
\]

(b) Obtain \( C_b \) factor [AISC F1].
From AISC Table 3-1, \( C_b = 1.30 \)

(c) Select beam section using "BEAM DESIGN MOMENTS", AISC 13th Ed. Manual, p. 3-127 using \( M_u(50/60)/C_b = 216(50/60)/1.30 = 38.5 \text{ ft-kips} \) with \( L_b = 14 \text{ ft} \). Find: W14x34;

The computed properties for W14x34 for \( F_y = 60 \text{ ksi} \) steel are: \( \phi_b M_p = 246 \text{ ft-kips}; \phi_b M_r = 153 \text{ ft-kips}; L_p = 4.93 \text{ ft}; L_r = 13.9 \text{ ft} \)

Since \( [L_b = 14 \text{ ft}] > L_r \), elastic lateral-torsional buckling equation applies:

\[
M_n = F_{cr} S_x \quad F_{cr} = \frac{C_b \pi^2 E}{(\frac{L_b}{t_b})^2} \sqrt{1 + \frac{0.078 J_e}{S_x h_o} (\frac{L_b}{t_b})^2}
\]

\[
F_{cr} = \frac{1.3 \pi^2 29000}{(14 \times 12)^2} \sqrt{1 + \frac{0.078 \times 0.569}{48.6(13.5)(1.8)^2}} = 53.85 \text{ ksi} \quad \phi M_n = 0.9(53.85)48.6
\]

\( \phi M_n = 2355 \text{ ft-lb} = 196.3 \text{ k-ft} \)

Thus, \( \phi_b M_n = 196.3 \text{ k-ft} \)

The flange and web local buckling limit states do not control because:

\[
\left[ \frac{b_f}{2t_f} = 7.4 \right] < \left[ \lambda_p = \frac{0.38 \sqrt{E}}{F_y} = 8.4 \right] \quad \left[ \frac{h}{t_w} = 43.1 \right] < \left[ \lambda_p = \frac{3.76 \sqrt{E}}{F_y} = 82.6 \right]
\]

Section
\[
F_y = 60 \quad Z_x \quad \phi_b M_p \quad \phi_b M_r \quad \phi_b M_n \quad M_u \quad \text{Corrected} \quad L_p \quad L_r \quad \frac{b_f}{2t_f} \quad \frac{h}{t_w}
\]

| W14x34 | 54.6 | 246 | 153 | 196 | 220 | 4.9 | 13.9 | 7.4 | 43.1 | NG |
| W16x36 | 64.0 | 288 | 198 | 222 | 220 | 4.9 | 13.7 | 8.12 | 48.1 | OK |

... use W16x36 \( F_y = 60 \text{ ksi} \) steel

(see calculation sheet on next page)
Computations for 9.1b and 9.2a

9.1b
\[ \tau_s = \frac{\sqrt{57.5(5960)}}{127} = 4.609 \rightarrow \tau_s = 2.147 \]
\[ J = 1.83, \quad h_0 = 20.4, \quad c = 1 \]
\[ L_r = 1.95(2.147) \frac{29000}{0.7(36)} \sqrt{1.83} \left( \frac{1.83}{127(20.4)} \right) \left[ 1 + \sqrt{1 + 6.76 \left( \frac{0.7(36)127(20.4)}{29000(1.83)} \right)^2} \right] \]
\[ L_r = 267.1" = 22.26' \]

9.2a
\[ W_{14\times3} = 54.6, \quad S_x = 48.6, \quad I_x = 1.53, \quad h_0 = 13.5 \]
\[ \phi M_p = 0.9(60) 54.6 = 2948.4 = 245.7 \text{ k-ft} \]
\[ \phi M_t = 0.9(0.7(48.6)60) = 1837 = 153.1 \text{ k-ft} \]
\[ L_p = 1.76(1.53) \frac{29000}{60} = 59.2" = 4.93', \quad J = 0.569 \]
\[ \tau_s = \frac{\sqrt{23.3(1070)}}{48.6} = 3.2489 \rightarrow \tau_s = 1.8025 \]
\[ L_r = 1.95(1.8) \frac{29000}{0.7(60)} \sqrt{0.569} \left( \frac{0.569}{48.6(1.53)} \right) \left[ 1 + \sqrt{1 + 6.76 \left( \frac{0.7(60)48.6(1.53)}{29000(0.569)} \right)^2} \right] \]
\[ L_r = 166.7" = 13.892' \]
\[ W_{16\times36} = 64, \quad S_x = 56.5, \quad I_x = 1.52, \quad h_0 = 1.83, \quad P_0 = 15.4, \quad J = 0.585, \quad C_0 = 1460 \]
\[ L_r = 1.95(1.83) \frac{29000}{0.7(60)} \sqrt{0.545} \left( \frac{0.545}{56.5(1.54)} \right) \left[ 1 + \sqrt{1 + 6.76 \left( \frac{0.7(60)56.5(15.4)}{29000(0.545)} \right)^2} \right] = 164.25" \]
\[ L_r = 13.69 \text{ k-ft} < L_b \]
\[ F_{cr} = \frac{1.3\pi^2}{29000} \left( \frac{14\times12}{1.83} \right) \left[ 1 + 0.078 \left( \frac{56.5(15.4)}{1.83} \right) \left( \frac{14\times12}{1.83} \right)^2 \right] = 52.46 \text{ ksf} \]
\[ \phi M_h = 0.9(52.46)56.5 = 2767 \text{ k-ft} \]
Select the lightest W section as a simply supported beam on a 35 ft span. The load is 0.2 kip/ft dead load plus the beam weight, and the live load is 0.6 kips/ft. Lateral supports exists at 7 ft intervals. Assume only flexure must be considered; i.e., omit treating shear and deflection. \( F_y = 65 \) ksi.

**Use LRFD**

(a) Obtain factored moment \( M_u \):
\[
M_u = \frac{1}{8} (1.20)(35)^2 = 184 \text{ ft-kips (w/o beam)}
\]

(b) Obtain \( C_b \) factor \([AISC F1]\).
From p.3-10 Table 3-1 \( C_b = 1.0 \text{ approx} \)  
(Correctly \( C_b = 1.008 \).

(c) Select beam section using "BEAM DESIGN MOMENTS", AISC Manual, p. 3-127 using \( M_u = 184(50/65) = 142 \text{ ft-kips with } L_b = 7 \text{ ft} \).
Assume \( \lambda \leq \lambda_p \); i.e., compact for flange and web local buckling.

Find: \( W14 \times 30, \phi_bM_n = 165 \text{ ft-kips} \)

From AISC p. 1-22 \( \frac{b c}{2 \varepsilon_p} = 8.74, \frac{p}{\varepsilon_w} = 45.4, S_x = 42, Z_x = 47.3 \)
\[
\phi M_p = 0.9 \left[ \frac{47.3}{65} \right] = 2767 \text{ k-in} = 230.6 \text{ k-ft} \\
\phi M_r = 0.9 \left[ \frac{42}{65} \right] = 1720 \text{ k-in} = 143.3 \text{ k-ft} \\
L_p = 1.76 \left( 1.49 \sqrt{\frac{29000}{65}} \right) = 55.39^\prime = 4.616 \text{ ft} \\
L_r = 1.95 \left( 1.77 \sqrt{\frac{29000}{0.7(65)}} \left[ \frac{0.38}{42 (13.5)} \right]^2 \right) = 4.616 \text{ ft} \\
L_r = 152.5^\prime = 12.71 \text{ ft} \quad \therefore \quad L_p < L_b < L_r
\]
\[
\phi M_n = 1.0 \left[ \frac{230.6 - (230.6 - 143.3)}{(12.71 - 4.616)} \right] = 204.9 \text{ k-ft} \\
\phi M_n = 204.9 \text{ k-ft} > M_u = 184 + 1.2(0.030)35^2 = 189.5 \text{ k-ft} \quad \text{OK}
\]
Check local buckling of flange p.16.1.16
\[
\lambda_p = 0.38 \sqrt{\frac{E}{\rho_y^4}} = 0.38 \sqrt{\frac{29000}{65}} = 8.03 < \frac{b f}{2 \varepsilon_p} = 8.74
\]
\( \therefore \) local buckling can be important.
\[ \lambda_f = 1.0 \sqrt{\frac{E}{F_y}} = 1.0 \sqrt{\frac{290000}{65}} = 21.12 \rightarrow \text{Eq (F3-1)} \]

\[
\phi M_n = \left[ \phi M_p - (\phi M_p - \phi M_r) \left( \frac{\lambda_f - \lambda_p}{\lambda_f - \lambda_r} \right) \right]
\]

\[
\phi M_n = \left[ 230.6 - (230.6 - 143.3) \left( \frac{8.74 - 8.03}{21.12 - 8.03} \right) \right] = 225.9 \text{ k-ft}
\]

Based on flange local buckling.

Since 225.9 k-ft > 204.9 k-ft \rightarrow \text{flange local buckling does not occur prior to lateral torsional buckling (LTB)}.

\[ \phi M_n = 204.9 \text{ k-ft} < M_u = 184 + 1.2 (0.030) \left( \frac{35}{8} \right)^2 = 189.5 \text{ k-ft} \text{ OK} \]

Use W14x30 \( F_y = 65 \text{ ksi} \) \text{ Steel}