All problems are to be done according to the AISC Load and Resistance Factor Design. Values of yield stress $F_y$ and tensile strength $F_u$ are available in AISC p. 2-39. Assume adequate lateral support of the compression flange such that lateral stability does not control. Be sure to check local buckling of all cross sectional elements.

7.1 Compute the nominal flexural strength $M_n$ of W12x14 $F_y=70$ ksi steel beam with full lateral support. See Table B4.1 in Chapter B of Specifications to check compactness with respect to flange and web. If non-compact, use Section F3 to compute $M_n$.

7.2 A simply supported welded I-section carries a concentrated load $W$ at midspan as shown in the figure. The load is 20% dead load and 80% live load. Determine the maximum service load $W$ that can be permitted under LRFD. Consider also the weight of the beam. Steel weighs 490 lbs/ft³. Assume $F_y=50$ ksi.

7.3 A simply supported beam with a span of 40 ft is to carry, in addition to its own weight, a uniformly distributed dead load of 0.4 k/ft, a uniformly distributed live load of 0.8 k/ft, and a concentrated live load of 15 kips at midspan as shown. The compression flange is assumed to have sufficient lateral support such that lateral stability does not control. Select the lightest W section using (a) A 572 Grade 50 Steel, (b) A572 Grade 65 Steel. Design is according to AISC Specifications Chapter F, Chapter B. See also steel beam selection Tables for $Z_x$ from AISC p. 3-11. Use compact sections for your design (i.e. $\lambda < \lambda_p$). See Table B4.1 in Chapter B for the definition of compactness with respect to flange and web.
W12 x 14  \( F_y = 70 \text{ ksi} \)  \( A = 4.16 \text{ in}^2 \)  \( Z_x = 17.4 \text{ in}^3 \)

\[ M_p = Z F_y = 17.4(70) = 1218 \text{ k-in} \]

Check Compactness \( \frac{bf}{2t_f} = 8.82 \), \( \frac{h}{t_w} = 54.3 \)

Check flange: Table B4.1 Case 1 \( \lambda_p = 0.38 \sqrt{\frac{E}{F_y}} \)

\[ \lambda_p = 0.38 \sqrt{\frac{29000}{70}} = 7.7345 < \frac{bf}{2t_f} \]

\( \lambda_r = 1.0 \sqrt{\frac{E}{F_y}} = 20.354 > \frac{bf}{2t_f} \)

\( \lambda_p < \frac{bf}{2t_f} < \lambda_r \)  Flange noncompact

Check web: Table B4.1 Case 9 \( \lambda_p = 3.76 \sqrt{\frac{E}{F_y}} \)

\[ \lambda_p = 3.76 \sqrt{\frac{29000}{70}} = 76.53 > \frac{h}{t_w} = 54.3 \]

\( \therefore \) Web is compact

Use Eq. (F3-1) to compute \( M_n \) with noncompact flange

\[ M_n = [M_p - (M_p - 0.7F_y S_x)(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}})] \quad (F3-1) \]

\[ S_x = 14.9 \text{ in}^3 \]

\[ M_n = [1218 - (1218 - 0.7(70)14.9)(\frac{8.82 - 7.7345}{20.354 - 7.7345})] = 1176 \text{ k-in} \]

\[ M_n = 1176 \text{ k-in} \approx 98 \text{ k-ft} \]
\[ F_y = 50 \text{ ksi} \]

\[ I_x = \frac{14(30)^3}{12} - \frac{13.5(29)^3}{12} = 4062.375 \text{ in}^4 \]

\[ Z_x = 2 \left[ \frac{1}{2} (14)(14.75) + \frac{1}{2} (14.5)(14.5) \right] \]

\[ S_x = \frac{I_x}{15} = 270,825 \text{ in}^3 \]

\[ M_p = Z_x F_y = 311.625(50) = 15581.25 \text{ k-in} \]

Check compactness of flange \( \frac{b_f}{2t_f} = \frac{14}{2(\frac{1}{2})} = 14 \)

Table B4.1 Case 2 \( \lambda_p = 0.38 \sqrt{\frac{E}{F_y}} \)

\[ \lambda_p = 0.38 \sqrt{\frac{29000}{50}} = 9.152 < 14 \rightarrow \text{noncompact} \]

\[ \lambda_r = 0.95 \sqrt{\frac{K_c E}{F_L}} \]

\[ K_c = \frac{4}{\sqrt{29(\frac{1}{2})}} = 0.5252 \quad F_L = 0.7 F_y \]

\[ \lambda_r = 0.95 \sqrt{\frac{0.5252(29000)}{0.7(50)}} = 19.81 > 14 \text{ ok not slender} \]

Check compactness of web \( \frac{b_w}{t_w} = \frac{29}{(\frac{1}{2})} = 58 \)

Table B4.1 Case 9 \( \lambda_p = 3.76 \sqrt{\frac{E}{F_y}} = 90.55 > 58 \rightarrow \text{web is compact} \)

Compute \( M_n \) by Eq. (E3-1)

\[ M_n = \left[ 15581 - (15581 - 0.7(50)270.8)(\frac{14 - 9.152}{19.81 - 9.152}) \right] = 12,805 \text{ k-in} \]

\[ M_n = 12,805 \text{ k-in} = 1,067 \text{ k-ft} \]
7.2 Continued

\[ \phi M_n = 0.9(1,067) = 960.3 \, k\text{-ft} \geq M_u \]

\[ M_u = \frac{WL}{4} + \frac{w_b L^2}{8} \quad ; \quad W = 20\%DL + 80\%LL \]

Compute weight of beam per unit length \( w_b \)

\[ w_b = \sum_b A \quad ; \quad A = 2(14)\frac{1}{2} + 29(\frac{1}{2}) = 28.5 \, \text{in}^2 \]

\[ w_b = 490 \, \# \frac{(28.5 \, \text{in}^2)}{(144 \, \text{in}^2)} = 96.98 \, \# \, \text{in/ft} = 0.09698 \, \text{k/ft} \]

\[ M_u = \left[ 0.20 W (1.2) + 0.80 W (1.6) \right] \frac{30}{4} + 0.09698 \frac{(30)^2}{8} \]

\[ M_u = 11.4 \, W + 10.91 \, k\text{-ft} \leq \phi M_n = 960.3 \, k\text{-ft} \]

\[ W \leq 83.28 \, k \]
7.3

\[ M_u = \frac{1.6(15)40}{4} + \frac{0.8(40)^2}{8} + \frac{1.2(0.4)40^2}{8} + \frac{1.2 W_b (40)^2}{8} \]

\[ M_u = 592 + 240 W_b \quad k-1 \]

(2) \( F_y = 50 \text{ ksi} \), assuming compact \( Z_x \), required \( \frac{Z_x}{\phi F_y} = \frac{M_u}{\phi F_y} \)

\[ Z_x \text{ required} > \frac{592(12)}{50(0.9)} = 157.9 \text{ in}^3 \]

From p. 3-16 \( W24 \times 68 \Rightarrow Z_x = 177 \text{ in}^3 \)

\[ M_u = 592 + 240(0.068) = 608.32 \quad k-1 \]

\[ \therefore Z_x \text{ required} = \frac{608.32(12)}{50(0.9)} = 162.2 < 177 \text{ OK} \]

\[ \therefore \text{Use } W24 \times 68 \text{ Grade 50} \]

(6) \( F_y = 65 \text{ ksi} \) \( \Rightarrow Z_x \text{ required} = \frac{608.32(12)}{(0.9)65} = 124.8 \text{ in}^3 \)

From p. 3-17 \( \Rightarrow W21 \times 55 \Rightarrow Z_x = 126 \text{ in}^3 > 124.8 \text{ OK} \)

Check compactness since \( F_y > 50 \text{ ksi} \)

Flange: Table B4.1 Case 1

\[ \lambda_p = 0.38 \sqrt{\frac{E}{F_y}} = 0.38 \sqrt{\frac{29000}{65}} = 8.03 > \frac{b_t}{2 t_f} = 7.87 \]

\[ \therefore \text{Flange is compact} \]

Web: Table B4.1 Case 9

\[ \lambda_p = 3.76 \sqrt{\frac{E}{F_y}} = 3.76 \sqrt{\frac{29000}{65}} = 79.42 > \frac{P_w}{t_w} = 50 \text{ ok} \]

\[ \therefore M_n = M_p \]

Use \( W21 \times 55 \text{ Grade 65 Steel} \)