All problems are to be done according to the AISC Load and Resistance Factor Design. Assume fastener strength is adequate and does not control. All holes are standard holes. Values of yield stress $F_Y$ and tensile strength $F_u$ are available in AISC 13th Ed. p. 2-39. Assume the slenderness ratio is desired to not exceed 300. Where needed, assume distances from center of hole to end of piece are 1-1/2 in. See Assignment 3 or Workshop 3 for gage distances in angle legs. For unequal leg angles assume the long leg is connected. All designs must be adequate for safety against gross section yielding, net section fracture, and block shear rupture limits.

4.1 Select the lightest single angle to support a tensile load. A single gage line of four bolts is to be used as shown. Assume 3-inch spacing of bolts along the direction of loading at first. Determine the actual required bolt spacing based on the block shear rupture strength. Dead Load=15 kips and Live Load=40 kips. Member is 18 ft long and bolt diameter is 3/4 in. Use A36 steel.

4.2 Select the lightest double angle tension member as shown to carry a load $P=200$ kips (30% DL and 70% LL). Use A572 Grade 50 steel and 7/8-inch diameter bolts. The length of the member is 15 ft. Initially assume 3-inch spacing of bolts along the direction of loading. Determine the actual required bolt spacing based on the block shear rupture strength. Assume a total of eight fasteners as shown.

4.3 Design lightest double angle shape for member BC of the truss shown. Assume 3/8-inch gusset plates. Use A572 Grade 50 Steel. The applied loads shown on the drawing are as follows: $P_1=20k$ DL and 40k LL, $P_2=10k$ DL and 12k LL, $P_3=10k$ LL. Assume a bolted connection with a single gage line of five 7/8-inch high strength bolts in standard size holes with end distance of 1.5-inches and center-to-center bolt spacing of 3 inches. Check gross section yielding, effective net section fracture, stiffness requirements, and block shear failure modes. If block shear strength controls, determine the necessary bolt spacing for sufficient block shear strength.
\[ \text{Pu} = 1.2 \text{DL} + 1.6 \text{LL} = 1.2(15) + 1.6(40) = 82 \text{ k} \]
\[ 82 \text{ k} \leq 0.9 \left(36\right) \text{Ag} \]
\[ 82 \text{ k} \leq 0.75 \left(58\right) \text{ Ae} \]
\[ \text{Ag required} = \frac{82}{0.9 \left(36\right)} = 2.531 \text{ in}^2 \]
\[ \text{Ae required} = \frac{82}{0.75 \left(58\right)} = 1.885 \text{ in}^2 \]
Assume \( U = 0.85 \) (to be checked later)
\[ \text{An required} = \frac{\text{Ae required}}{U} = \frac{1.885}{0.85} = 2.218 \text{ in}^2 \]
\[ \Gamma_{\text{min required}} = \frac{L}{300} = \frac{18 \left(12\right)}{300} = 0.72 \text{ in.} \]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( t_d )</th>
<th>( \frac{\text{Ae required} + td}{U} )</th>
<th>Single Angle Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{4} \left(\frac{7}{8}\right) )</td>
<td>( 0.21875 ) ( 1.885 ) ( 0.21875 ) ( 2.436 )</td>
<td>need ( Ag = 2.531 \text{ in}^2 ) there are none ( V_d )</td>
</tr>
<tr>
<td>( \frac{5}{16} )</td>
<td>( \frac{5}{16} \left(\frac{7}{8}\right) )</td>
<td>( 0.27344 ) ( 1.885 ) ( 0.27344 ) ( 2.491 )</td>
<td>need ( Ag = 2.531 \text{ in}^2 ) ( L5 \times 3\frac{1}{2} \times \frac{5}{16} \rightarrow Ag = 2.56 \text{ in}^2 ) ( \bar{X} = 0.829 \text{ in} ) ( U = 1 - 0.829 = 0.971 ) ( U &gt; 0.85 ) ( \text{OK} )</td>
</tr>
<tr>
<td>( \frac{3}{8} )</td>
<td>( \frac{3}{8} \left(\frac{7}{8}\right) )</td>
<td>( 0.3281 ) ( 1.885 ) ( 0.3281 ) ( 2.546 )</td>
<td>need ( Ag = 2.546 \text{ in}^2 ) ( L4 \times 3\frac{1}{2} \times \frac{3}{8} \rightarrow Ag = 2.67 \text{ in}^2 ) ( \bar{X} = 0.947 \text{ in} ) ( U = 1 - 0.947 = 0.053 ) ( 0.8948 &gt; 0.85 ) ( \text{OK} )</td>
</tr>
</tbody>
</table>

\( \because \ L5 \times 3\frac{1}{2} \times \frac{5}{16} \text{ A36 is the lightest section with sufficient strength.} \)

Check stiffness: \( \Gamma_e = 0.758 > 0.72 = \Gamma_{\text{min required}} \text{ OK} \)
Check Block Shear Rupture for L5 x 3\(\frac{1}{2}\) x 5\(\frac{5}{16}\)

\[
\frac{\alpha_v}{2''} = \frac{0.5}{3} = 3 \Rightarrow 3''
\]

\[
\alpha_v = [0.5 - 3.5(\frac{7}{8})] = 2.3242
\]

\[
\alpha_t = [2 - 0.5(\frac{7}{8})] = 0.4883
\]

\[
\alpha_y = 10.5(\frac{5}{16}) = 3.28125
\]

\[
0.6 F_v \alpha_v = 0.6(58)2.3242 = 80.88 \quad \text{shear fracture does not control}
\]

\[
F_u \alpha_t = 58(0.4883) = 28.32
\]

\[
0.6 F_y \alpha_y = 0.6(36)3.28125 = 70.88 \quad \text{shear yielding more critical}
\]

\[
\phi R_h = 0.75 [70.88 + 28.32] = 74.40 \quad k < 82 \quad \text{NG}
\]

\[\frac{\alpha_y}{2''} = \frac{0.6(36)(5 + 1.5)5}{16} + 28.32\]

\[
82k \leq 0.75 [0.6(36)(3.5 + 1.5)]
\]

\[
s \geq 3.5''
\]

Check

\[
\phi R_h = 0.75 [0.6(36)(12)5 + 28.32] = 82k \quad \text{OK}
\]

\[\frac{\alpha_v}{2''} = \frac{1.5}{3.5} = 3.5 \Rightarrow 3.5''
\]

Use bolt spacing \(s = 3\frac{1}{2}''\) for block shear

L5 x 3\(\frac{1}{2}\) x 5\(\frac{5}{16}\) A36
4.2

\[ P_u = 1.2DL + 1.6LL = 200(0.30) 1.2 + 200(0.70) 1.6 = 296 \text{ kN} \]

\[ 296 \text{ kN} \leq 0.9(50) A_g \]

\[ 296 \text{ kN} \leq 0.75(65) A_e \]

\[ A_g \text{ required} = \frac{296}{0.9(50)} = 6.578 \text{ in}^2 \]

\[ A_e \text{ required} = \frac{296}{0.75(65)} = 6.072 \text{ in}^2 \]

\[ \text{required} = \frac{1}{300} = \frac{15(12)}{300} = 0.6 \]

\[ A_e = U A_n \text{ (AISC Chapter D specs.)} \]

\[ U = 1 - \frac{x}{L_c} \]

\[ L_c = 3 \times 3 = 9" = \text{length of connection} \]

\[ U = 0.9 \text{ or } U = 0.85 \text{ can be tentatively assumed (check)} \]

Effective hole diameter \( d = \frac{7}{8} + \frac{1}{8} = 1 \text{ in.} \)

Assume \( U = 0.85 \) to start the design iteration, we will compute \( U \) after selecting trial section.

The total area of 2L required based on gross section yielding was \( A_g = 6.578 \text{ in}^2 \); based on net section fracture, the required area is

\[ A = A_n + 4td = \frac{A_e}{U} + 4td = 2 \text{ holes per angle}. \]

\[ A = \frac{6.072}{0.85} + 4t(1) \]

\[ A = 7.144 + 4t \]
4.2 Continued (1): \( A_{g, \text{required}} = 6.578 \text{ in}^2 \); \( A_e = 6.072 \text{ in}^2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( 4td )</th>
<th>( \frac{A_e}{U} + 4td )</th>
<th>Double Angle Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{4} )</td>
<td>1.0</td>
<td>( \frac{6.072}{0.85} + 1 = 8.14 \text{ in}^2 )</td>
<td>none</td>
</tr>
<tr>
<td>( \frac{5}{16} )</td>
<td>1.25</td>
<td>8.394 \text{ in}^2</td>
<td>none</td>
</tr>
</tbody>
</table>
| \( \frac{3}{8} \) | 1.5 | 8.644 \text{ in}^2 | \( 6 \times 6 \times \frac{3}{8} \rightarrow A = 2(4.38) \)
\[ = 8.76 \text{ in}^2 \]
\[ U = 1 - \frac{1.62}{9} = 0.82 \]
\[ 8.90 > 8.76 \text{ NG} \]
\[ L = \frac{15(12)}{1.87} = 96.26 < 300 \text{ OK} \]
| \( \frac{1}{2} \) | | \[ \frac{6.072}{0.82} + 1.5 = 8.90 \] | \[ 7 \times 4 \times \frac{7}{16} \rightarrow A = (4.63)^2 \]
\[ A = 9.26 \text{ in}^2 \]
\[ \text{OK but slightly overdesigned} \]

2L - 6x6 x 3/8 Grade 50 steel is the lightest section. It does not work with the given connection length. However, we may need to increase the connection length to satisfy block shear. The minimum value of \( U \) is obtained from \( \frac{6.072}{0.85} + 1.5 = 8.76 \rightarrow U_{\text{min}} = 0.836 \)
\[ 0.836 = 1 - \frac{1.62}{\ell} \rightarrow \ell = 9.9'' \text{ is required for connection length to be adequate with regard to net section fracture}. \]

Check slenderness w/r \( r_e \) of each angle
\[ \frac{L}{r_e} = \frac{15(12)}{1.19} = 151. < 300 \text{ - OK - No intermediate filler plate is needed}. \]
4.2 Continued (2)  

Check block shear per angle  \( A = 4.38 \) 

\[ A_{nt} = \left(3.75 - 1.5\right) \frac{3}{8} = 0.8438 \text{ in}^2 \]

\[ A_{nu} = \left(10.5 - 3.5\right) \frac{3}{8} = 2.625 \text{ in}^2 \]

\[ F_{uA_{nt}} = 65 \left(0.8438\right) = 54.85 \text{ k} \]

\[ 0.6F_{uA_{nu}} = 0.6 \left(65\right) = 39.75 = 118.125 \text{ k} \]

\[ \phi R_n = 0.75 \left[102.38 + 54.85\right] = 117.92 \text{ k} < \frac{296}{2} = 148 \text{ k} \]

\[ 148 \text{ k} = \phi R_n = 0.75 \left[0.675 \left(38 + 1.5 - 3.5\right) \frac{3}{8} + 54.85\right] \]

\[ 142.48 = 43.875 \text{ k} \]

\[ S = \frac{171.73}{43.875} = 3.914'' \text{ bolt spacing needed} \]

To prevent block shear failure under the factored loads, use 8 = 4''

```
+1.5'' + 4'' + 4'' + 4'' +
```

Use 2L-6x6x3/8 Grade 50 Steel with the shown bolt spacing.
4.3 Compute factored loads
\[ P_1F = 1.2(20) + 1.6(40) = 88 \text{k} \]
\[ P_2F = 1.2(10) + 1.6(12) = 31.2 \text{k} \]
\[ P_3F = 1.6(10) = 16 \text{k} \]

Analyze truss under factored loads:

FBD of entire truss: \( \Sigma M_D = 0 \)
\[ 38(16) + 88(32) - A_y(48) + 31.2(40) + 62.4(24) + 31.2(8) - 16(12) = 0 \]
\[ 48 A_y = 7027.2 \quad \rightarrow \quad A_y = 146.4 \text{k} \]

FBD ABE: \( \Sigma M_F = 0 \)
\[ F_{BC}(12) + 31.2(16) - 16(12) - 146.4(24) + 88(8) = 0 \]
\[ F_{BC}(12) - 2502.4 = 0 \]
\[ F_{BC} = 208.53 \text{k} = P_u \]

Factored Tension Load
4.3 Continued

\[ 208.53 \leq 0.9(50)A_g \rightarrow A_g = \frac{208.53}{0.9(50)} = 4.634 \text{ in}^2 \text{ required} \]
\[ 208.53 \leq 0.75(65)A_e \rightarrow A_e = \frac{208.53}{0.75(65)} = 4.278 \text{ in}^2 \text{ required} \]

\[ \Gamma_{\text{required}} = \frac{L}{300} = \frac{16(12)}{300} = 0.64 \]

\[ U = 1 - \frac{X}{L} = 1 - \frac{X}{9} \quad \text{Assume } U = 0.85 \text{ initially; } d = \frac{7}{8} + \frac{1}{8} = 1.0'' \]

<table>
<thead>
<tr>
<th>t</th>
<th>2td</th>
<th>( \frac{A_e}{U} + 2td )</th>
<th>Double Angle Option</th>
</tr>
</thead>
</table>
| 5/16 | 0.625 | \( \frac{4.278}{0.85} + 0.625 = 5.658 \) | \( 6 \times 3\frac{1}{2} \times \frac{5}{16} \rightarrow A = 2(2.87) = 5.74 \)
\[ U = 1 - \frac{0.763}{9} = 0.92 \]
\[ \Gamma_e = 0.768 \rightarrow \frac{L}{\Gamma_e} = 2.50 < 300 \text{ OK} \]
| 3/8  | 0.75  | \( \frac{4.278}{0.90} + 0.75 = 5.503 \) | \( 2L - 4 \times 4 \times \frac{3}{8} \rightarrow A = 2(2.86) = 5.72 \text{ in}^2 \)
\[ U = 1 - \frac{1.13}{9} = 0.87 \]
\[ \Gamma_e = 0.779 \quad \text{OK} \]
\[ 2L - 5 \times 3 \times \frac{3}{8} \rightarrow A = 2(2.86) = 5.72 \text{ in}^2 \]
\[ U = 1 - \frac{0.698}{9} = 0.92 \]
\[ \Gamma_e = 0.646 \rightarrow \frac{L}{\Gamma_e} = 2.97 < 300 \text{ OK} \]

Use \( 2L - 6 \times 3\frac{1}{2} \times \frac{5}{16} \) (LLBB) \{ A572 Grade 50 \}

or \( 2L - 4 \times 4 \times \frac{3}{8} \)

or \( 2L - 5 \times 3 \times \frac{3}{8} \) (LLBB)

2L - 6 \times 3\frac{1}{2} \times \frac{5}{16} \ is slightly heavier than the other two choices. Therefore, 2L - 4 \times 4 \times \frac{3}{8} \ and 2L - 5 \times 3 \times \frac{3}{8} \ are the lightest choices.
Check Block Shear for 4x4 x 3/8 - 2L

\[ A_{nt} = (1.5 - 0.5) \frac{3}{8} = 0.375 \text{ in}^2 \]
\[ A_{nv} = (13.5 - 4.5) \frac{3}{8} = 3.375 \text{ in}^2 \]
\[ A_{qv} = (13.5) \frac{3}{8} = 5.0625 \]

\[ F_u A_{nt} = 65(0.375) = 24.375 \text{ k} \quad 0.6 F_u A_{qv} = 0.6(50)5.0625 = 151.875 \text{ k} \]

\[ 0.6 F_u A_{nv} = 0.6(65)3.375 = 131.625 \text{ k} \]

\[ \phi R_n = 0.75 \left[ 131.625 + 24.375 \right] = 117 \text{ k/angle} \]

2L → 2(117) = 234k > 208.53k OK

Block Shear Strength is Sufficient

Checking Block Shear for 5x3 x 3/8 - 2L is unnecessary because Ant is larger than that for 4x4 x 3/8.

\[ A_{nt} = (2 - 0.5) \frac{3}{8} = 0.5625 \]
\[ A_{nv} = (13.5 - 4.5) \frac{3}{8} = 3.375 \]
\[ A_{qv} = (13.5) \frac{3}{8} = 5.0625 \]

\[ F_u A_{nt} = 65(0.5625) = 36.56 \text{ k} \quad 0.6 F_u A_{qv} = 131.625 \text{ as above} \]
\[ \phi R_n = 0.75 \left[ 131.625 + 36.56 \right] = 126.14 \text{ k/angle} \]

2L - 5x3 x 3/8 → \[ \phi R_n = 2(126.14) = 252.3 \text{ k} > 208.53 \text{ k} \]

Block Shear Strength is Sufficient
Check Block Shear for 2L-6 x 3½ x 5/16

\[ \text{Ant} = (2.5 - 0.5) \frac{5}{16} = 0.625 \text{in}^2 \]
\[ \text{Aqv} = 13.5 \left( \frac{5}{16} \right) = 4.21875 \]
\[ \text{Anv} = (13.5 - 4.5) \frac{5}{16} = 2.8125 \]

\[ F_u \text{Ant} = 65(0.625) = 40.625 \text{k} \]
\[ 0.6 F_u \text{Anv} = 0.6(65)2.8125 = 109.6875 \]

\[ 0.6 F_u \text{Aqv} = 0.6(50)4.21875 = 126.56 \]

\[ \phi R_n = 0.75 \left[ 109.6875 + 40.625 \right] = 112.73 \text{k/angle} \]

2L-6 x 3½ x 5/16 \[ \phi R_n = 2(112.73) = 225.47 \text{k} > 208.53 \text{k} \]

Block shear strength is sufficient for 2L-6 x 3½ x 5/16 also

We note that if we had only four bolts instead of 5 in the connection block shear strength would not be sufficient and we would have to increase the bolt spacing to satisfy the block shear requirements.

The next two pages show the process by which we would determine the required length if block shear strength controlled.
4.3 Continued: If we had only 4 bolts instead of 5

Check Block Shear: $A_{gr} = 10.5 \left(\frac{5}{16}\right) = 3.28125 \text{ in}^2$

$6 \times 3 \frac{1}{2} \times \frac{5}{16}$

$A_{nt} = 2.5 \left(\frac{5}{16}\right) - 0.5 \left(\frac{5}{16}\right) = 0.625 \text{ in}^2$

$A_{nv} = (10.5 - 3.5) \left(\frac{5}{16}\right) = 2.1875 \text{ in}^2$

$F_u A_{nt} = 65(0.625) = 40.625 \text{ k}$

$0.6 F_u A_{nv} = 0.6(65)2.1875 = 85.31 \text{ k}$

$0.6 F_u A_{gr} = 0.6(65)3.28125 = 98.44 \text{ k}$

$\Phi R_n = 0.75 \left[85.31 + 40.625\right] = 94.45 \text{ k/angle}$

N.G. we need $\frac{208.53}{2} = 104.27 \text{ k/angle}$

$104.27 = 0.75 \left[(x - 3.5) \left(\frac{5}{16}\right) (0.6)65 + 40.625\right]$ (see x on figure)

$73.80 = (x - 3.5) 9.141 \rightarrow x = 11.57''$

'' Increase bolt spacing to $\frac{11.57 - 1.5}{3} = 3.36''$ to prevent block shear rupture for $6 \times 3\frac{1}{2} \times \frac{5}{16}$ LLBB.

---

Check Block Shear for $4 \times 4 \times 3\frac{3}{8}$ L

$1.5$

$2.5$

$x$

$A_{nt} = (1.5 - 0.5) \left(\frac{3}{8}\right) = 0.375 \text{ in}^2$

$A_{nv} = (10.5 - 3.5) \left(\frac{3}{8}\right) = 2.625 \text{ in}^2$

$F_u A_{nt} = 65(0.375) = 24.375 \text{ k}$

$0.6 F_u A_{nv} = 0.6(65)2.625 = 102.375 \text{ k}$

$\Phi R_n = 0.75 \left[102.375 + 24.375\right] = 95.06 \text{ k/angle}$

N.G. we need $104.27 \text{ k/angle}$

$104.27 = 0.75 \left[(x - 3.5) \left(\frac{3}{8}\right) (0.6)65 + 24.375\right]$\n
$114.65 = (x - 3.5) 14.625 \rightarrow x = 11.34''$

Increase bolt spacing to $\frac{11.34 - 1.5}{3} = 3.28''$ to prevent block shear rupture for $\frac{3}{8} L = 4 \times 4 \times 3\frac{3}{8}$
Check block shear for $5 \times 3 \times 3/8$-

\[ A_{nt} = (2-0.5)\frac{3}{8} = 0.5625 \text{ in}^2 \]
\[ A_{nv} = (10.5-3.5)\frac{3}{8} = 2.625 \text{ in}^2 \]
\[ F_a A_{nt} = 65(0.5625) = 36.56 \text{k} \]

\[ 0.6 F_y A_{gu} = 118.13 \text{k} \text{ (as before); } \]
\[ 0.6 F_y A_{nv} = 102.375 \text{k} \text{ (as before) } \]

\[ \phi R_n = 0.75 \left[ 102.375 + 36.56 \right] = 104.203 \text{k/angle} \]
\[ \phi R_n = 104.2 \text{k/angle} \approx 104.27 \text{k/angle} \text{ OK} \]

No need to increase bolt spacing for L - $5 \times 3 \times 3/8$. Therefore 2L - $5 \times 3 \times 3/8$ is the most economical design because it requires the shortest connection length.