Some Elementary Aspects of Boundary Layer Theory

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The failure of potential flow theory to predict drag on objects when a fluid flows past them provided the impetus for Prandtl to put forward a theory of the boundary layer adjacent to a rigid surface. Prandtl's principal assumptions are listed below.

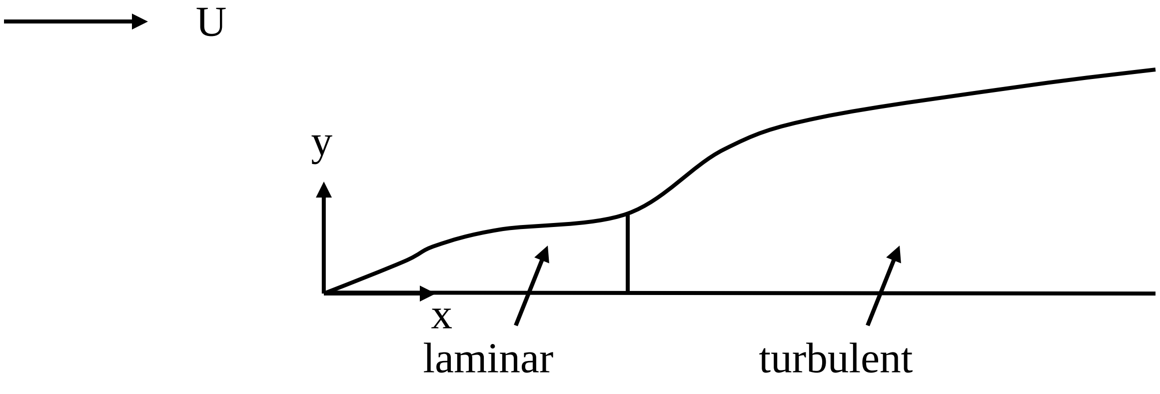
Assumptions

1. When a fluid flows past an object at large values of the Reynolds number, the flow region can be divided into two parts.

(i) Away from the surface of the object, viscous effects can be considered negligible, and potential flow can be assumed.

(ii) In a thin region near the surface of the object, called the boundary layer, viscous effects cannot be neglected, and are as important as inertia.

2. The pressure variation can be calculated from the potential flow solution along the surface of the object, neglecting viscous effects altogether, and assumed to be impressed upon the boundary layer.



Transition from laminar to turbulent flow in the boundary layer on a flat plate occurs at $\mathrm{Re}_x \approx 5\times10^5$, where $\mathrm{Re}_x = \dfrac{xU}{\nu}$. Here, ν is the kinematic viscosity of the fluid.

The following approximate estimates can be written for the thickness of laminar and turbulent boundary layers.

Laminar Boundary Layer

$$\frac{\delta}{x} \sim \frac{5}{\sqrt{\mathrm{Re}_x}}$$

Turbulent Boundary Layer

$$\frac{\delta}{x} \sim \frac{0.16}{\left(\mathrm{Re}_x\right)^{1/7}}$$

Some aspects of the drag on a body when a fluid flows past it at high Reynolds number

From the four videos by Professor A. Shapiro, we learned some important ideas about the drag experienced by a body when a fluid flows past it. Professor Shapiro showed experimental observations that initially appeared to be paradoxical, and later explained them using physical aspects of boundary layer theory.

1. The drag on a sphere at first increases as the velocity of the fluid is increased. But at a certain value of the velocity, the drag suddenly decreases drastically, and then begins to increase again as the velocity is subsequently increased. We learned that this is because the boundary layer at first is laminar at small velocities, and separates from the sphere immediately past the shoulder, creating a large wake behind the sphere in which reverse flow vortices are maintained by the external flow. A simplistic explanation of boundary layer separation is given here. For the correct explanation for the formation of a standing eddy behind a blunt object, caused by accumulation of vorticity, and the consequent flow separation, you should consult Section 4.12 of Batchelor (1967). In this picture, the formation of reverse flow eddies is the cause of flow separation, not the consequence of it. Nevertheless, a simplistic explanation is provided below.

It is suggested that boundary layer separation occurs because the pressure gradient from potential flow is impressed on the boundary layer fluid. In the forward region facing the flow, the pressure gradient is favorable, with pressure decreasing from the forward stagnation point toward the shoulder. But, in the rear of the object, the pressure gradient is adverse. This means that the pressure increases in the direction of motion of fluid in the boundary layer. The only mechanism for the boundary layer to resist this adverse pressure gradient is the exchange of momentum from the free stream. This is a weak mechanism for a laminar boundary layer. The adverse pressure gradient causes the boundary layer to separate from the body, and leads to a recirculation in the wake. The pressure in the wake is relatively low when compared with the pressure in the forward half of the sphere. This pressure difference pushes the sphere in the direction of flow. As a consequence there is a large “pressure drag” on the sphere, in addition to “skin friction drag,” a term used to describe drag from the shear stress exerted on the surface of the sphere by the flow. The total drag is the sum of the skin friction drag and the pressure drag. As the velocity is increased gradually, at a certain value, the flow in the boundary layer becomes turbulent. This leads to turbulent exchange of momentum with the free stream which is far more efficient than viscous transport of momentum. Therefore, the free stream is able to transfer a significant amount of momentum to a turbulent boundary layer in the rear half of the sphere, causing the boundary layer to stay attached farther. This means that the wake is much smaller, with a consequent reduction in the pressure drag. Even though the skin friction drag is increased slightly, the reduction in the pressure drag is much larger, and leads to a net reduction in the drag on the sphere when the boundary layer becomes turbulent. Eventually, as the free stream velocity is increased further, the concomitant increase in the skin friction drag leads to a higher drag on the sphere.

2. Professor Shapiro showed us that the drag on a slightly roughened sphere can be smaller than the drag on a smooth sphere in a certain range of velocities. This can be explained by noticing that the boundary layer becomes turbulent at lower velocities on the slightly roughened sphere than on the smooth sphere. This leads to advancement of the point of separation behind the rough sphere, with a reduced wake, and consequently reduced pressure drag. As a result, the rough sphere experiences less drag than a smooth sphere over a certain range of velocities.

3. Professor Shapiro demonstrated that the drag on a streamlined object is much smaller than that on a blunt object that offers the same projected shape to the flow when flow occurs at high Reynolds number past these objects,

even though the streamlined object has a much larger surface area. In contrast, at low Reynolds number, the streamlined shape experiences larger drag. These facts can be explained using the concept of the boundary layer and its separation behind the blunt object in high Reynolds number flow. The streamlined object is designed to minimize and nearly eliminate boundary layer separation in the region of adverse pressure gradient. Therefore, even though the skin friction drag on it is larger than that on the blunt object, the pressure drag is nearly eliminated, leading to a smaller total drag than that on a blunt object. In contrast, at low Reynolds number, there is no boundary layer, and the drag is caused only by viscous forces at the surface of the object. Streamlining offers no special advantage here, and the drag is larger on the streamlined object because it has a much larger surface area than the blunt object used in the experiments.

References

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