

- 1) Given that $n(t)$ is a Gaussian White noise process with a constant spectrum S_o , $R_{nn}(\tau) = 2\pi S_o \delta(\tau)$, and

$$\frac{d^3 Y}{dt^3} + \frac{d^2 Y}{dt^2} - \frac{dY}{dt} - Y = n(t) + \frac{dn}{dt}$$

- a) Evaluate the power spectrum of Y process, $S_{YY}(\omega)$.
 b) Evaluate $R_{YY}(\tau)$, $E\{Y^2\}$, $E\{Y^6\}$.
- 2) Suppose V is a uniform random variable with

$$f_V(v) = \begin{cases} 1 & 0 \leq v \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

We define two stochastic processes, $X(t) = u(t-V)$, $Y(t) = \delta(t-V)$. (Assume $0 \leq t \leq 1$.)

- a) Evaluate the expected value of X and Y.
 b) Evaluate the $R_{XX}(t_1, t_2)$, $R_{YY}(t_1, t_2)$, $R_{XY}(t_1, t_2)$. (Assume $0 \leq t_1 \leq 1$ and $0 \leq t_2 \leq 1$.)
- 3) Let

$$Z(t) = \int_0^t \tau^2 X(\tau) d\tau$$

with $R_{XX}(t_1, t_2) = |t_1 - t_2|$. Find $E\{Z^2(t)\}$.

- 4) Given that

$$\begin{aligned} \frac{dX}{dt} + Y &= n(t) \\ \frac{dY}{dt} - X &= \frac{dn}{dt} \end{aligned}$$

- a) Find $S_{YY}(\omega)$, $S_{XX}(\omega)$, $R_{XX}(\tau)$ and $R_{YY}(\tau)$.
 b) Evaluate $E\{X^2\}$, and $E\{X^{250}\}$.
 c) Find the expression for the probability density function for random process X(t).

Table of Fourier Transform

$f(t)$	$\bar{f}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t) dt$
$e^{-\alpha t }$	$\frac{2\alpha}{\omega^2 + \alpha^2}$
$e^{-\alpha t } \cos(\omega_o t)$	$\frac{2\alpha(\alpha^2 + \omega_o^2 - \omega^2)}{(\alpha^2 + \omega_o^2 - \omega^2)^2 + 4\alpha^2 \omega^2}$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$\cos(\omega_o t)$	$\pi\{\delta(\omega + \omega_o) + \delta(\omega - \omega_o)\}$
$e^{-\alpha t } \left\{ \cos(\beta t) + \frac{\alpha}{\beta} \sin(\beta t) \right\}$	$\frac{4\alpha(\alpha^2 + \beta^2)}{(\omega^2 - \alpha^2 - \beta^2)^2 + 4\alpha^2 \omega^2}$
$e^{-\alpha t } (1 + \alpha t)$	$\frac{4\alpha^3}{(\omega^2 + \alpha^2)^2}$
$\int_{-\infty}^{+\infty} f(t - \tau) g(\tau) d\tau$	$\bar{f}(\omega) \bar{g}(\omega)$
$\begin{cases} 1 & t \leq T \\ 0 & t > T \end{cases}$	$2T \frac{\sin(\omega T)}{\omega T}$
$\frac{\sin(\omega_o t)}{\pi}$	$\begin{cases} 1 & \omega \leq \omega_o \\ 0 & \omega > \omega_o \end{cases}$
$e^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha} e^{-\omega^2 / 4\alpha^2}$
$\begin{cases} 1 - \frac{ t }{T} & t \leq T \\ 0 & t > T \end{cases}$	$4 \frac{\sin^2(\omega T / 2)}{T \omega^2}$