Quantum dynamics in nonequilibrium strongly correlated environments

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We consider a biased quantum point contact between two Luttinger liquids measuring a quantum-mechanical system (oscillator). There are three main physical results. First, we find that for voltages higher than the oscillator level spacing, the effect of the measurement is equivalent to coupling to a heat bath, with an effective temperature that only depends on the device *I-V* characteristic. Second, we find that this heating changes the *I-V* characteristics on the junction. Finally, we predict a modulation of the tunneling current at the oscillator frequency. Also, we find that a generalized nonequilibrium fluctuation-dissipation relation connects the decoherence and dissipation of the oscillator to the current-voltage characteristics of the device.

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I. INTRODUCTION

The quest to build a scalable quantum computer has recently led to a series of spectacular experiments where macroscopic quantum states were coherently manipulated and measured.[1–3] These experiments for the first time give an opportunity to study the effects of indirect continuous measurement[4] on an individual quantum system. Largely motivated by the technological breakthroughs, a detailed theoretical analysis of measurement process based on explicit models describing the coupling between a quantum system and an electrical measurement apparatus has been initiated. In most cases, strong and weak measurement of two- (spin) or a few-level systems have been considered.[5–9] It has been found that in the case of the weak measurement, when the timescale associated with the measurement is slow compared to the intrinsic dynamics of the quantum system, it is possible to indirectly observe the coherent behavior of the system.[9] In the opposite limit of the strong measurement, due to the fast dephasing, the dynamics of the quantum system can be frozen (quantum Zeno effect), an effect particularly useful in single-shot qubit readout.[6,9] To date, however, there have not been many studies of quantum-measurement of systems with infinite number of levels. Examples of particular practical importance are quantum mechanical vibrational modes, be it man-made resonators or isolated molecular/atomic phonon modes. The applications include the local probes, such as atomic force microscopy and magnetic-resonance force microscopy, as well as novel nano-electronic devices where an imbedded phonon mode can provide new functionality.[10] A study of nano-scale mechanical devices[11] is also intriguing, since it may help to explore the fundamental problem of the measurement-induced quantum to classical crossover.[12]

Interactions and correlations play a crucial role in protecting quantum coherence in macroscopic systems and enabling manipulation of the quantum states. In submicrometer electronic systems, the Coulomb interaction becomes important and can lead to Coulomb blockade, a subject of intensive research both in the contexts of "classical" and quantum-coherent electronic devices. The more subtle effects of *itinerant* electron-electron interactions, however, have not been studied in the context of quantum measurement. These effects are often important since in order to interface with a quantum device, at least a part of the apparatus has to be scaled down to the device size. For instance, carbon nanotubes are actively considered as one of the basic elements for nanocircuitry. Due to essentially one-dimensional confinement, the electron-electron interactions in the nanotubes lead to dramatic renormalizations near the Fermi surface, attributed to the formation of the Luttinger liquid.[13] The study of such correlation effects is, therefore, important in developing the understanding of a realistic quantum measurement.

To analyze the role of correlations in the measurement apparatus, we study here a specific example of two Luttinger liquid leads electrically coupled to a quantum system, with the tunneling current being influenced by a coordinate of the system. The interactions in the Luttinger liquids can be parametrized by the dimensionless constant g, which describes repulsive interactions if $g<1$, attractive if $g>1$, and a noninteracting Fermi liquid for $g=1$. One experimental realization of repulsive Luttinger liquids is carbon nanotubes,[14] where it has been shown that defects in the nanotubes play the role of tunnel contacts. We mainly deal here with the repulsive case, and the attractive Luttinger liquids are discussed at the end of the paper.

We begin by developing the formalism for an arbitrary quantum system, which extends that of Mozyrsky and Martin,[12] as it is valid for general leads as well as for arbitrary voltage with the weak tunneling regime as the only approximation. There are three main physical results. First, we find that for higher voltages the effect of the measurement is equivalent to coupling to a heat bath, with an effective temperature T_{eff} different from the Fermi-liquid case.[12] We find that for tunneling particles of charge q across a junction with given instrument I-V characterististic,

$$T_{\text{eff}}=(qI/2)(dV/dI). \tag{1}$$

For Luttinger liquids with $g>1$, this reduces to $T_{\text{eff}}=(\alpha+1)^{-1}qV/2$, where the current $I\propto V^{\alpha+1}$ and α is the sum of density of state exponents for the two leads: $\alpha(g_1)+\alpha(g_2)$. For carbon nanotubes, tunneling between the end of two leads is given by $\alpha_{\text{end}}(g)=(1/g-1)/4$,[22] while for

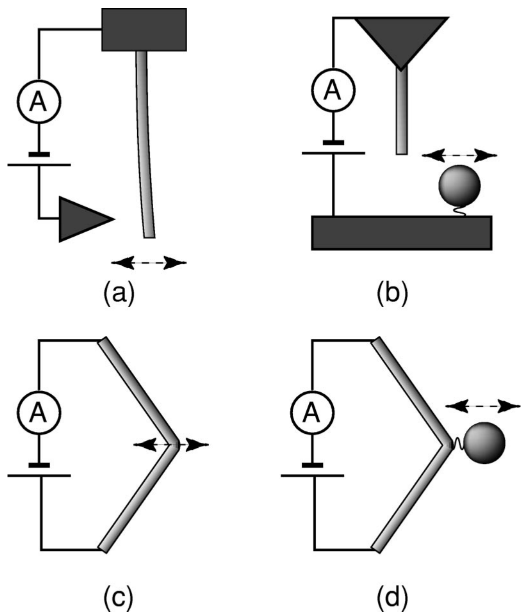

FIG. 1. Experimental realizations of the model.

tunneling between two infinite leads one has $\alpha_{\rm tun}=(g+g^{-1}-2)/8$. Second, we find that this heating changes the I-V characteristics of the junction [Eq. (13)]. Third, we observe a modulation of the tunneling current at the oscillator frequency [Eq. (15)]. We finally considered the possible case of attractive interactions, which can be handled via a dual representation: here, a weak backscattering approximation is used rather than a weak tunneling approximation, and all the results carry through with the roles of voltage and current interchanged.

Examples of experimental realizations of our model as applied to the measurement of a quantum oscillator are shown in Fig. 1. In Figs. 1(a) and 1(b), the tunnel junction is formed by a nanotube (Luttinger liquid) and a metal (Fermi liquid), while in Figs. 1(c) and 1(d) both sides of the junction are Luttinger liquids. In the first example, Fig. 1(a), the tunnel current between the gate and the nanotube is used to monitor the transverse nanotube oscillations. The characteristic oscillation frequency is about 1 GHz for a 100-nm nanotube,[15] which makes it possible to achieve the quantum regime at about 50 mK. In Fig. 1(b), a short stiff nanotube in the STM mode[16] is used to perform vibrational spectroscopy of an adsorbate loosely bound to a metal surface. The position of the atom/molecule modulates the tunnel current. In the last two examples, a kink in the nanotube formed either mechanically or due to a 5-7 defect plays the role of the tunnel contact.[14,17] The presence of the defect will lead to formation of a localized optical-phonon mode above the nanotube phonon band [above 200 meV (Ref. 18)], which will couple to the tunnel current by modifying the tunneling matrix element. Alternatively, the chemical kink defect can be functionalized by adsorbing an atom or molecule,[19] whose vibrations will also modify the tunneling between the two Luttinger legs.

II. KELDYSH FORMALISM

We establish a general formalism for effects of measurement in strongly correlated environments. We use as an example the specific problem of tunneling between two Luttinger liquids, when the tunneling is coupled to an external system, such as a quantum oscillator, or a spin. The Hamiltonian is

$$\mathcal{H}=\Omega T+\mathcal{H}_1^L+\mathcal{H}_2^L+\mathcal{H}_0, \tag{2}$$

where $\mathcal{H}_0$ is the Hamiltonian for the measured system, referred to as an oscillator, $\mathcal{H}^L_{1,2}$ are the Luttinger liquid Hamiltonians for the two leads, with Luttinger parameter g and with a potential difference $\mu=qV$. We define the electron-tunneling operator $T(t)=\Psi_1^\dagger(x=0,t)\Psi_2(x=0,t)+\text{H.c.}$, where $\Psi_{1,2}(x,t)$ are fermion operators in the leads. The term Ω includes c-number terms as well as operators that do not commute with $\mathcal{H}_0$.

Following Kane and Fisher,[17] we consider tunneling via a weak link, and consider repulsive interactions $g>1$ to avoid infrared divergences (the case of attractive interactions is considered later in a dual representation). We use a Keldysh formalism.[20] Our procedure closely follows the one used to study noise in Luttinger liquid tunneling.[21] Let us suppose that initially the oscillator and leads are decoupled, with density matrices ρ_0 for the oscillator and $\rho^L_{1,2}$ for the leads, so that the full density matrix $\rho=\rho_0\otimes\rho_1^L\otimes\rho_2^L$. For now, we assume that the leads are at zero temperature. After the interaction between systems is turned on at time $t=-\infty$, the systems become coupled. Define the scattering operator $\mathcal{S}$ by

$$\mathcal{S}=T_c\exp\left(-i\int_\infty^{-\infty}\mathcal{H}dt\right)\exp\left(-i\int_{-\infty}^{\infty}\mathcal{H}dt\right)\equiv 1, \tag{3}$$

where the operator T_c denotes time ordering along the Keldysh contours. Points along the forward branch $(-\infty\to\infty)$ are ordered with increasing times, while those in the return $(\infty\to-\infty)$ are ordered with decreasing times, with those in the return branch ordered after those in the forward branch. We will occasionally use a superscript f,r on t to indicate to which contour t belongs. The expectation value of any product of operators $O(t_1),O(t_2),\cdots$ can be obtained by $\langle O(t_1)O(t_2)\cdots\rangle=\text{Tr}\{\rho(T_cO(t_1^f)O(t_2^f)\cdots\mathcal{S})\}$, where we work in the Schrödinger representation throughout.

It is well established since the work of Kane and Fisher[17] that for repulsive interactions, a renormalization-group calculation leads to no infrared divergences and a perturbative approach remains valid for *all* voltages. Thus, we can trace out the Luttinger liquids and to obtain the new scattering operator to order Ω^2,

$$\begin{aligned}\mathcal{S}_{\rm eff}=T_c&\exp\left(-i\int_\infty^{-\infty}\mathcal{H}_0dt\right)\exp\left(-i\int_{-\infty}^{\infty}\mathcal{H}_0dt\right)\\&\times\exp\left(\pm\frac{1}{2}\int_{-\infty}^{\infty}dt_1dt_2\Omega(t_1)\Omega(t_2)\right.\\&\left.\times\frac{2\cos(\mu(t_1-t_2))}{(\epsilon\pm i(t_2-t_1))^{\alpha+2}}+\cdots\right),\end{aligned} \tag{4}$$

where the plus sign is chosen before the integral over t_1,t_2 if t_1,t_2 are in different branches and the minus sign is chosen if they are in the same branch. The $\pm$ sign in the denominator of the exponential is taken positive if t_2 is after t_1 and negative if t_2 is before t_1. We have used $\langle T(t_1)T(t_2)\rangle_L = 2\cos[\mu(t_1-t_2)]/[\epsilon\pm i(t_2-t_1)]^{\alpha+2}$, where the expectation value $\langle\rangle_L$ for the leads is the expectation value for decoupled Luttinger liquids at zero temperature.[21] In this expectation value, there is an additional factor dependent on the density of states, which may be absorbed into the normalization of Ω. The $\cdots$ in Eq. (4) denote terms of higher order in Ω. This is the only approximation in the present formalism, a weak tunneling approximation that $\Omega^2\mu^\alpha \ll 1$.

The expectation value of an operator $O(t)$ becomes

$$\langle O(t)\rangle = \mathrm{Tr}\{\rho_0(T_c\tilde{O}(t)\mathcal{S}_{\mathrm{eff}})\} = \langle\tilde{O}(t)\rangle, \tag{5}$$

where $\tilde{O} = \langle O(t_1^f)\rangle_L \mp \int dt\, i\Omega(t)\langle O(t_1^f)T\rangle_L + \cdots$. Here the ellipses denote connected expectation values which are of higher order in Ω, and where $\mp$ is chosen negative for t on the forward contour and positive for t on the return contour. The operator $\tilde{O}$ depends only on the oscillator coordinates and not on the leads.

The exponential in Eq. (4) involves a product $\Omega(t_1)\Omega(t_2)$, where t_1, t_2 may be on either the forward or return contour. We introduce $\Omega^s(t_1) = \Omega(t_1^f) + \Omega(t_1^r), \Omega^a(t_1) = \Omega(t_1^f) - \Omega(t_1^r)$. Then,

$$\begin{aligned}
&\pm\Omega(t_1)\Omega(t_2)\frac{2\cos[\mu(t_1-t_2)]}{[\epsilon\pm i(t_2-t_1)]^{\alpha+2}}\\
&\quad = -\Omega^a(t_1)\Omega^a(t_2)\mathrm{Re}\left(\frac{2\cos[\mu(t_1-t_2)]}{(\epsilon+i|t_2-t_1|)^{\alpha+2}}\right)\\
&\qquad -2\Omega^a(t_1)\Omega^s(t_2)\theta(t_1-t_2)\mathrm{Im}\left(\frac{2\cos[\mu(t_1-t_2)]}{(\epsilon+i|t_2-t_1|)^{\alpha+2}}\right).
\end{aligned} \tag{6}$$

Again assuming that $\Omega^2\mu^\alpha \ll 1$, we can make a further simplification in the exponential of Eq. (4), by using the Bloch-Redfield approximation,[23] that $\Omega^{a,s}(t_2) = e^{i\mathcal{H}_0(t_2-t_1)}\Omega^{a,s}(t_1)e^{-i\mathcal{H}_0(t_2-t_1)}$. Corrections to the Bloch-Redfield approximation arise if operators Ω are inserted between t_1, t_2; such corrections to $\mathcal{S}_{\mathrm{eff}}$ will be of the order of $|t_2-t_1|\Omega^2\mu^{\alpha+1}$. Let us write $\Omega = \Omega_{ij}$, where i,j denote eigenstates of $\mathcal{H}_0$ with energies $E_{i,j}$. Then, integrating over t_2,

$$\begin{aligned}
\mathcal{S}_{\mathrm{eff}} = {}& T_c\exp\left(-i\int_\infty^{-\infty}\mathcal{H}_0 dt\right)\exp\left(-i\int_{-\infty}^{\infty}\mathcal{H}_0 dt\right)\\
&\times\exp\left(\frac{1}{2}\int_{-\infty}^{\infty}dt[\Omega^a(t)\Omega^s_{ij}(t)A(\mu,E_i-E_j)\right.\\
&\left.-\Omega^a(t)\Omega^a_{ij}(t)S(\mu,E_i-E_j)]\right),
\end{aligned} \tag{7}$$

where we define $S(\mu,\Delta E) = \int_{-\infty}^{\infty}2\cos(\mu t)e^{i\Delta Et}\mathrm{Re}[(\epsilon + i|t|)^{-\alpha-2}]dt$, $A(\mu,\Delta E) = -2\int_{-\infty}^{0}2\cos(\mu t)e^{i\Delta Et}\mathrm{Im}[(\epsilon + i|t|)^{-\alpha-2}]dt$. The terms in $\Omega^a\Omega^a$ in Eq. (7) produce decoherence, and are equivalent to averaging over a randomly fluctuating field coupled to Ω, while the terms $\Omega^a\Omega^s$ produce dissipation.

Taking $|E_i-E_j| < \mu$, so that the correct poles in the integrals for A,S are determined by the sign of μ, one finds that

$$\begin{aligned}
S(\mu,\Delta E) &= \frac{1}{2}[I(\mu+\Delta E)+I(\mu-\Delta E)]A(\mu,\Delta E)\\
&= \frac{1}{2}[I(\mu+\Delta E)-I(\mu-\Delta E)]+\cdots,
\end{aligned} \tag{8}$$

where the ellipses denotes imaginary terms, possibly singular as $\epsilon\to 0$, which may be absorbed into a renormalization of $\mathcal{H}_0$, and hence dropped. We have defined $I(\mu) = 2\pi\mu^{\alpha+1}/\Gamma(\alpha+2)$.

III. AVERAGE CURRENT AND NOISE

Here $qI(\mu)$ is equal to the current[24] flowing at $\Omega=1$. We now recompute the current within the present formalism, in order to obtain corrections to the current due to fluctuations in Ω. The current operator at time $t^f=0$ is $J(0^f) = qi\Omega(0^f)[\Psi_1^\dagger(0,0^f)\Psi_2(0,0^f) - \mathrm{H.c.}]$. From Eqs. (5), the leading contribution to $\langle J\rangle$ is of the order of Ω^2, $\pm iq\int_{-\infty}^{\infty}dt\langle\Omega(t)\Omega(0^f)\rangle 2\sin(\mu t)/(\epsilon\pm it)^{\alpha+2}$. This vanishes when integrated over t on the forward contour, so we can assume that t is on the reverse contour. Applying the same Bloch-Redfield approximation, we get $q\int_{-\infty}^{\infty}dt e^{i(E_i-E_j)t}\langle\Omega_{ij}(0)\Omega(0^f)\rangle 2\sin(\mu t)/(\epsilon+it)^{\alpha+2}$. Doing this integral yields

$$\langle J\rangle = q\langle\Omega_{ij}(0)\Omega(0)\rangle\frac{2\pi}{\Gamma(\alpha+2)}(\mu+E_i-E_j)^{\alpha+1}. \tag{9}$$

We now consider fluctuations in the current, $\langle J(t_1^r)J(t_2^f)\rangle$. The order Ω^2 contribution to the current-current correlation function, from Eq. (5), is given by

$$q^2\langle\Omega(t_1^r)\Omega(t_2^f)\rangle\frac{2\cos[\mu(t_2-t_1)]}{(\epsilon+i|t_2-t_1|)^{2+\alpha}}. \tag{10}$$

This represents the shot noise in the tunnel junction slightly modulated by oscillator. To next order in Ω^2, from Eq. (5), we must compute

$$\begin{aligned}
&-\pm\pm(q^2/2)\int_{-\infty}^{\infty}dt_3dt_4\langle\Omega(t_1^r)\Omega(t_2^f)\Omega(t_3)\Omega(t_4)\rangle\\
&\quad\times\langle T(t_1^r)T(t_2^f)T(t_3)T(t_4)\rangle_L.
\end{aligned}$$

Even for c number Ω, this calculation is involved;[21] in this case, the calculation yields a result of the order the $\mu^{2\alpha}\Omega^4|t_2-t_1|^{-2}$, plus terms with lower powers of μ. However, for operator Ω, there is one contribution which is of the order of $\mu^{2\alpha+2}$ and which dominates over the previous contribution for $|t_2-t_1| \gg \mu^{-1}$, the time regime we now consider.

The expectation value $\langle T(t_1^r)T(t_2^f)T(t_3)T(t_4)\rangle_L = \langle T(t_1^r)T(t_3)\rangle_L\langle T(t_2^f)T(t_4)\rangle_L + t_3\leftrightarrow t_4 + \text{connected}$. The last term, a connected expectation value of four T operators, gives the contribution of the order of $\mu^{2\alpha}\Omega^4|t_2-t_1|^{-2}$. We ignore this, and consider only the other terms. Integrating over t_3, t_4, and applying a Bloch-Redfield approximation, we arrive at the following contribution to $\langle J(t_1)J(t_2)\rangle$:

$$\pm\pm\langle\Omega(t_1^r)\Omega_{ij}(t_1)\Omega(t_2^f)\Omega(t_2)\rangle \times\left[\int dt_3\langle T(t_1^r)T(t_3)\rangle_L e^{i(E_i-E_j)(t_3-t_1)} \times\int dt_4\langle T(t_2^f)T(t_4)\rangle_L e^{i(E_k-E_l)(t_4-t_2)}\right]. \quad (11)$$

For the expectation value $\langle T(t_1^f)T(t_3)\rangle_L\langle T(t_2^f)T(t_4)\rangle_L$ to be significant, $t_1-t_3\sim\mu^{-1}$, $t_2-t_4\sim\mu^{-1}$, justifying the application of the Bloch-Redfield approximation. As claimed, Eq. (11) is of the order of $\mu^{2\alpha+2}$, reflecting a modulation of the current[7] by the oscillator. Equation (11) decays on a time scale of the order of the inverse damping coefficient, $1/\gamma$, of the oscillator. In the case of c number Ω, this term is neglected: when computing $\langle JJ\rangle-\langle J\rangle\langle J\rangle$, it cancels.

IV. DENSITY MATRIX

For $\Delta E\ll\mu$, it is possible to write the results above in a compact form. Define $\rho(t)$ to be the density matrix for the oscillator at given time t. Then use Eqs. (5) and (7) to compute $\langle\dot{\tilde{O}}(t^f)\rangle$ for any operator $\tilde{O}$; the result is a linear differential equation for the expectation values. Then use $\langle\dot{\tilde{O}}(t^f)\rangle\equiv\mathrm{Tr}[\dot{\rho}(t)\tilde{O}]$ to derive the equation for the density matrix:

$$\dot{\rho}=-i[\mathcal{H}_0,\rho]+\frac{1}{2}\frac{dI}{d\mu}[\Omega,\{\Lambda,\rho\}]-\frac{I}{2}l[\Omega,[\Omega,\rho]], \quad (12)$$

with $\Lambda=[\mathcal{H}_0,\Omega]$. Comparing to the results for Fermi-liquid leads,[12] the effective temperature of the oscillator, determined by the ratio of the third (decoherence) term to the second (dissipation) term in Eq. (12), is $T_{\mathrm{eff}}=(\alpha+1)^{-1}qV/2$. For the current, we find $\langle J\rangle=q\langle\Omega^2\rangle I(\mu)+q\langle\Lambda\Omega\rangle dI/d\mu$.

V. SPECTRAL REPRESENTATION

While these results were derived for Luttinger liquid leads, they are more general. Assuming sufficiently small Ω, we find Eq. (4) with $2\cos[\mu(t_1-t_2)]/[\epsilon\pm i(t_2-t_1)]^{\alpha+2}$ replaced by the appropriate expectation value in the leads, $\langle T(t_1)T(t_2)\rangle_L$. Let the density of states (particle and hole, respectively) at energy E be $\rho_{1,2}^{p;h}(E)$ in leads 1, 2, respectively. Then, defining $\rho^p(E)=\int dE_1\rho_1^p(E_1)\rho_2^h(E-E_1)$, $\rho^h(E)=\int dE_1\rho_1^h(E_1)\rho_2^p(E-E_1)$, the expectation value is $\int dE[\rho^p(E)e^{\mp i(E+\mu)(t_1-t_2)}+\rho^h(E)e^{\mp i(E-\mu)(t_1-t_2)}]$, with the minus sign chosen if t_1 is after t_2 and the plus sign otherwise. Then going through the same steps, we find Eqs. (7) and (8) in general, with $I(\mu)$ replaced by the appropriate current-voltage characteristic of the device: $I(\mu)=2\pi[\rho^p(-\mu)+\rho^h(\mu)]$. The relation between S, A, and I generalizes the fluctuation-dissipation relation derived between noise and current.[25] For $\Delta\ll\mu$, we arrive at Eq. (1).

VI. APPLICATIONS

We now consider the specific case of a harmonic oscillator with frequency ω_0 and mass m, linearly coupled to the tunneling, $\Omega=\Omega_0+cx$. We consider a regime for which the Ω_0 term dominates and the oscillator coordinate x only weakly modulates the current. For $\mu>\omega_0$, the average current is

$$J(\mu)=q\left[I\Omega_0^2+\frac{c^2S^2}{2m\omega_0A}-\frac{c^2A}{2m\omega_0}\right], \quad (13)$$

where $I=I(\mu);S=S(\mu,\omega_0);A=A(\mu,\omega_0)$. The first term gives $J\propto\Omega_0^2V^{\alpha+1}$, while the second term is proportional to $c^2\langle x^2\rangle V^{\alpha+1}\sim V^{\alpha+2}$. For very large V, the tunneling approximation breaks down, while for small V the first contribution dominates; however, for sufficiently large c at intermediate V, the second contribution is dominant.

The noise spectrum can be evaluated from Eqs. (10) and (11). Fourier transforming Eq. (10) gives the shot noise contribution

$$q^2\int dte^{i\omega t}\langle\Omega(0)\Omega(t)\rangle_L\langle T(0)T(t)\rangle_L=2q^2\Omega_0^2I, \quad (14)$$

for $|\omega|\ll|\mu|$. The above current modulation yields to leading order in c^2, after symmetrizing in t_1,t_2,

$$\frac{q^2\Omega_0^2c^4}{m^2}\frac{(I+S)^2S-2A^2(S+I)}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2}, \quad (15)$$

where $\gamma=c^2A(\mu,\omega_0)/(m\omega_0)$. At the peak, the signal-to-noise ratio, Eq. (15) divided by Eq. (14), is $\sim 2I^2/A^2$.

VII. ATTRACTIVE INTERACTIONS

We finally return to the case of an *attractive* interaction between electrons, tunneling between the ends of two leads so that $\alpha<0$. In this case, the density of states $\rho(E)$ diverges at small E in the ends of the leads. This gives rise to low-energy divergences in the perturbative expansion in the tunneling, as the tunneling operator becomes relevant in the attractive case. Thus, following Kane and Fisher,[17] we consider another possible system for which perturbative expansions are possible, that of a single lead with a weak backscattering. In this dual case, the Hamiltonian is

$$\mathcal{H}=\tilde{\Omega}U+\mathcal{H}^L+\mathcal{H}_,, \quad (16)$$

where $\mathcal{H}_0$ remains the oscillator Hamiltonian, $\mathcal{H}^L$ is the Hamiltonian for a *single* Luttinger liquid, U is a scattering potential at the link, $U=\Psi^\dagger(x=0)\Psi(x=0)$, and $\tilde{\Omega}$ is some function of the oscillator coordinate.

In the case of attractive interactions, the backscattering operator U is irrelevant in the dual problem, and a perturbative expansion is again possible. Thus, given that the weak tunneling fixed point is unstable, while the weak backscattering point is stable, it is reasonable to assume that the renormalization-group (RG) flow joins the two perturbative regimes, so that for attractive interactions the low-energy behavior is described by Hamiltonian (16), even if the original problem involves two separate leads. Conversely, for repulsive interactions, the weak tunneling description will always be correct at low energy. In this context, low energy means that both ω_0 and V are small compared to the bandwidth of the leads. In the Fermi-liquid case, $g=0$, both the weak tunneling and weak backscattering problems give rise to marginal behavior, and either leads to a convergent perturbation theory, with the choice between the two descriptions to be made on physical grounds.

In this RG flow, the operator $\tilde{\Omega}$ is not necessarily related to the original operator Ω. However, if the original operator Ω involves a weak linear coupling to the oscillator: $\Omega=\Omega_0+cx$ for small c, then $\tilde{\Omega}$ will also involve a linear coupling, $\tilde{\Omega}=\tilde{\Omega}_0+\tilde{c}x$, since no symmetry principle forbids the linear coupling to x in $\tilde{\Omega}$.

Starting with Hamiltonian (16), the problem can be solved as above, where the perturbation theory in T is replaced by a perturbation in U. After integrating out the lead, we arrive again at action (4), but with $\alpha=(g-1)/2$, rather than $\alpha=(1/g-1)/2$ as it would be previously. Then, for repulsive interactions, with $g>1$, we still have $\alpha>0$ and the perturbation theory works. In the dual problem, the roles of current and voltage are interchanged. Thus, defining $\mu(I)$ to be the voltage drop for given current at $\Omega=1$, we have $\mu(I)=2\pi I^{\alpha+1}/\Gamma(\alpha+2)$. The dynamics of the density matrix of the oscillator is described by Eq. (12), with the roles of μ and I interchanged, thus solving the attractive case as well.

VIII. DISCUSSION

We have developed a formalism for studying measurement of tunneling in strongly correlated systems. This formalism can be applied to several experimental setups using carbon nanotubes. We predict a heating of localized modes at the tunnel contact with a universal temperature depending only on I-V characteristics. Physically, this heating is due to the electron shot noise at the tunnel contact, and is modified from the Fermi-liquid case due to strong correlations. The heating can be measured as a correction to the I-V characteristics of the device, which may affect the experimental determination of the Luttinger parameter g from the I-V measurements. The most direct measurement of these modes is as a predicted peak in the power spectrum of the current at the mode frequency ω_0.

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