

Clarkson University  
Department of Mechanical & Aeronautical Engineering  
**Mechanics of Composite Materials**  
**AE 457, CE 421/521, ME 457/557**  
Spring 2004 – Exam II  
Tuesday, April 6, 2004

Name: \_\_\_\_\_

Student ID No.: \_\_\_\_\_

**Instructions:**

Please check now and make sure your exam is complete. There should be 8 pages including this coversheet, with 5 problems. Page 7 provides properties and stiffness matrices for carbon/epoxy lamina. Page 8 is a formula sheet.

Read the problems carefully and plan your time so as to gain the maximum number of points. The exam has 5 problems and is worth 100 points – some problems are worth more than others. You will be given partial credit only for work that is leading to a logical conclusion.

Please write neatly and clearly. Cross out any work that you do not wish to be considered.

**This exam is open book and closed notes, however you are permitted to have a single sheet of notes and a calculator.**

You may remove the staple and/or attach extra pages of work, but make sure that you staple all the exam pages and extra work pages together before turning in the exam. Missing pages cannot be accepted after the exam.

Grade:

1. \_\_\_\_\_/16

4. \_\_\_\_\_/25

2. \_\_\_\_\_/18

5. \_\_\_\_\_/25

3. \_\_\_\_\_/16

Total: \_\_\_\_\_/100

1. (16 points) For a  $[-45/0/45/90]$  laminate made of AS4/3501-6 carbon/epoxy (properties given on page 7). Calculate  $B_{xx}$ . Report your final answers in units of N (Newtons).

2. (18 points) For a  $[30/-30/30/-30]$  laminate made of AS4/3501-6 carbon/epoxy (properties given on page 7), determine the following:

a) What type of laminate is this?

b) Which terms of the laminate stiffness matrix,  $[D]$ , are zero?

c) Calculate the value of  $D_{xy}$ . Report your final answers in units of  $N \cdot m$ .

3. (16 points) For a  $[15/-15/-15/15]$  laminate made of AS4/3501-6 carbon/epoxy (properties given on page 7), determine the following:

a) What type of laminate is this?

b) Calculate the laminate major Poisson's ratio,  $\bar{\nu}_{xy}$ .

4. (25 points) A  $[30/0/-30]$  made of AS4/3501-6 carbon/epoxy (properties given on page 7). The laminate is cured at  $150^\circ\text{C}$  and cooled to a temperature of  $50^\circ\text{C}$ . There is no change in moisture. The coefficients of thermal expansion are:

$$\alpha_1 = 0$$

$$\alpha_2 = 27 \times 10^{-6} / ^\circ\text{C}$$

Determine resulting the hygrothermal force,  $N_y^{HT}$ . Report your final answer in units of N/m. (Note:  $\cos 30^\circ = \sqrt{3}/2$ ,  $\sin 30^\circ = 1/2$ )

5. (25 points) A  $[0/90]$  carbon/epoxy (AS4/3501-6) laminate (properties given on page 7) is cured at  $145^\circ\text{C}$  and cooled to a temperature of  $45^\circ\text{C}$ . There is no change in moisture. The coefficients of thermal expansion are:

$$\alpha_1 = -0.9 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha_2 = 27 \times 10^{-6} / ^\circ\text{C}$$

Due to the temperature change, the following reference plane strains and curvatures were determined:

$$\begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_s^o \end{bmatrix} = \begin{bmatrix} -632 \\ -632 \\ 0 \end{bmatrix} \times 10^{-6} \qquad \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix} = \begin{bmatrix} +11.8 \\ -11.8 \\ 0 \end{bmatrix} \text{m}^{-1}$$

Determine the residual stress,  $\sigma_{xe}$  at the midpoint of the  $90^\circ$  ply. (Assume the  $90^\circ$  ply to be the bottom ply and the  $0^\circ$  ply to be the top ply). Present final answer in units of MPa.

## Properties of carbon/epoxy (AS4/3501-6)

$$E_1 = 142 \text{ GPa}$$

$$E_2 = 10.3 \text{ GPa}$$

$$G_{12} = 7.2 \text{ GPa}$$

$$\nu_{12} = 0.27$$

$$\nu_{21} = 0.02$$

Ply thickness =  $t = 0.1 \text{ mm}$

$$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 143 & 3 & 0 \\ 3 & 10 & 0 \\ 0 & 0 & 7 \end{bmatrix} \text{ GPa}$$

$$\theta = 15^\circ \rightarrow \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix}_{\theta=15^\circ} = \begin{bmatrix} 127 & 10 & 29 \\ 10 & 12 & 4 \\ 29 & 4 & 15 \end{bmatrix} \text{ GPa}$$

$$\theta = 30^\circ \rightarrow \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix}_{\theta=30^\circ} = \begin{bmatrix} 87 & 25 & 42 \\ 25 & 21 & 16 \\ 42 & 16 & 29 \end{bmatrix} \text{ GPa}$$

$$\theta = 45^\circ \rightarrow \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix}_{\theta=45^\circ} = \begin{bmatrix} 47 & 32 & 33 \\ 32 & 47 & 33 \\ 33 & 33 & 37 \end{bmatrix} \text{ GPa}$$

$$\theta = 60^\circ \rightarrow \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix}_{\theta=60^\circ} = \begin{bmatrix} 21 & 25 & 16 \\ 25 & 87 & 42 \\ 16 & 42 & 29 \end{bmatrix} \text{ GPa}$$

## Units

$$\text{GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2, \quad \text{MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2, \quad 1 \text{ mm} = 10^{-3} \text{ m}$$