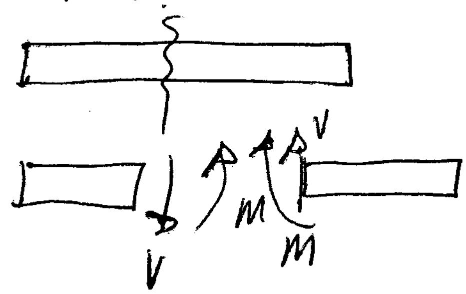
Exam III Review Session

Chpt. 5 - Shear + Bending moment diagrams

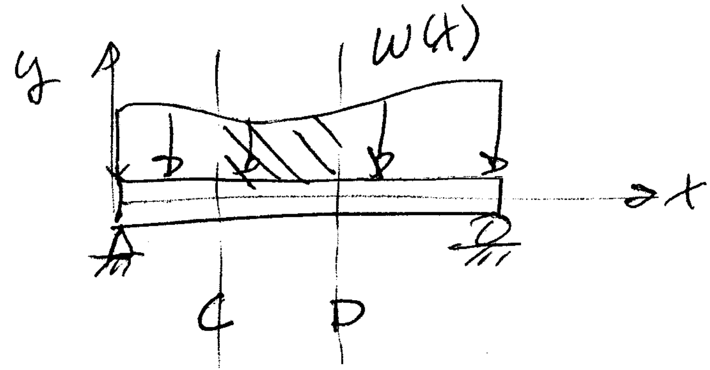
Determine internal shear force, bending moment due to applied loads

Sign convention:



Positive V, M

Beam:



$$\frac{dV}{dx} = -w$$

$$\frac{dM}{dx} = +V$$

$$V_D = V_C - \int_{x_C}^{x_D} w(x)\,dx = V_C - \text{(area under load curve)}$$

$$M_D = M_C + \int_{x_C}^{x_D} V(x)\,dx = M_C + \text{(area under shear curve)}$$

→ does not include concentrated forces or moments! → these are "jumps" in V, M diagrams

W	V	M
0	0	Constant
0	constant	linear
constant	linear	parabola
linear	parabola	cubic

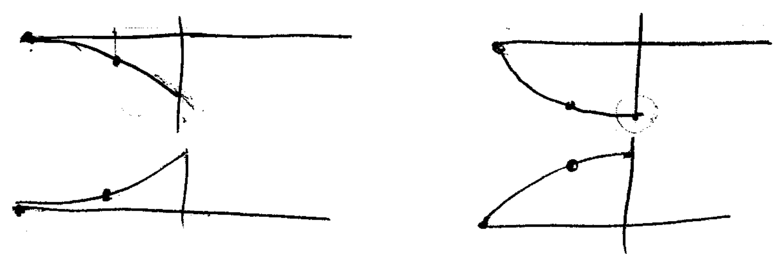
Other tips :

- Slope of shear curve = −w
- slope of moment curve = V

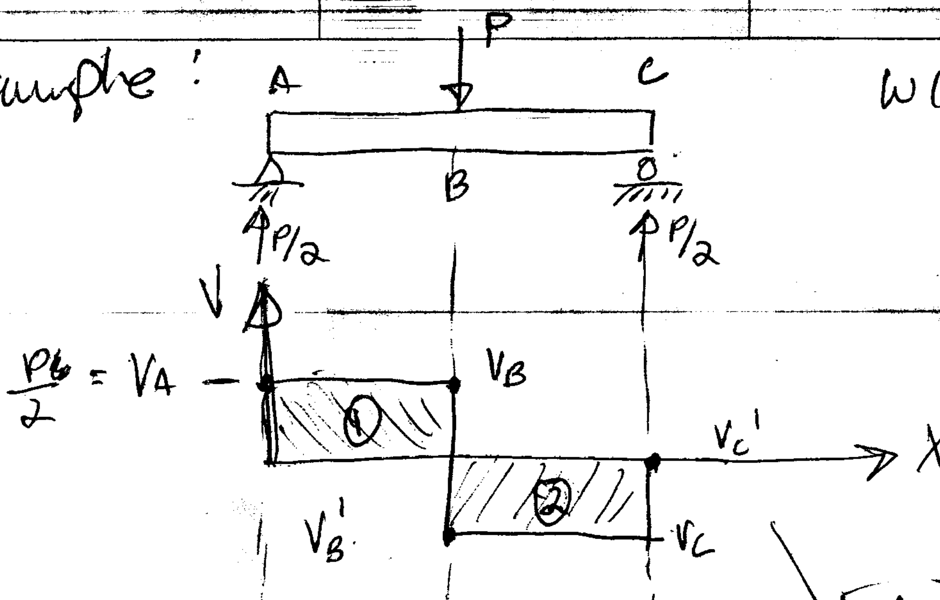
  V=0 → min max in M curve

Use these to decide how to sketch parabola or cubic

- If V or M is parabola or cubic − check point midway to see whether curve is :



Example: $w(x) = 0$



AB: V is constant so M is linear

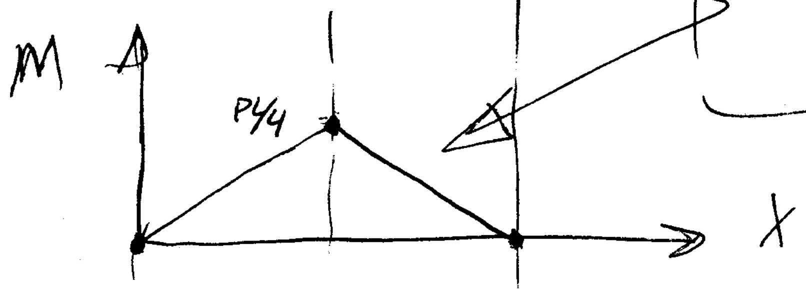
BC: V is constant so M is linear

$$V_B = V_A - \int \cancel{w}\,dx = V_A = P/2$$

$$V_B' = V_A - P = -P/2$$

$$V_C = V_B' - \int \cancel{w}\,dx = V_B' = -P/2$$

$$V_C' = V_B' + P/2 = 0$$



$M_A = 0$ (pinned)

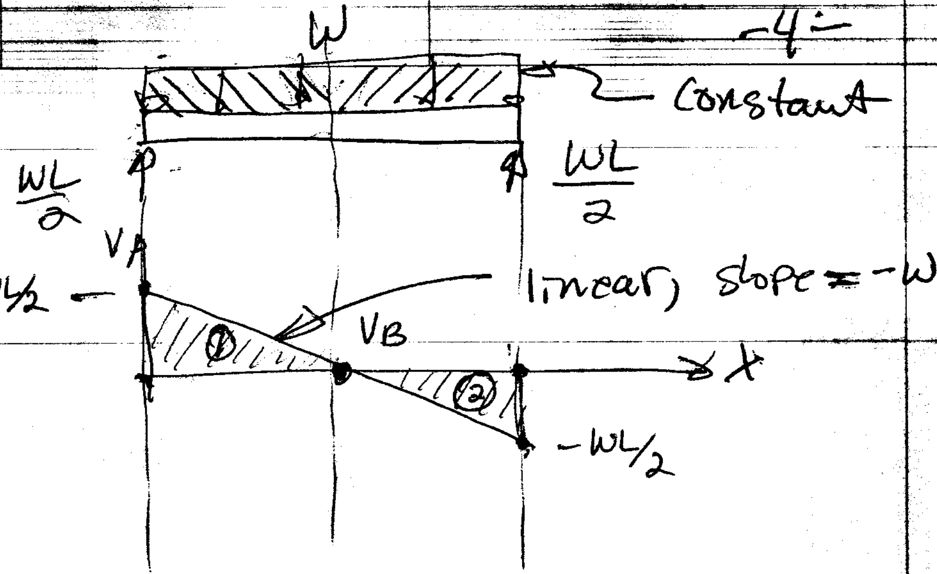
$$M_B = M_A + ① = 0 + \frac{P}{2}\left(\frac{L}{2}\right) = PL/4$$

$$M_C = M_B + ② = \frac{PL}{4} - \frac{P}{2}\left(\frac{L}{2}\right) = 0$$

Note: ① + ② = 0 ✓ since $M_A = M_C = 0$

$M_C = M_A + ① + ② = 0$ ✓

Example:



$\backslash\backslash\backslash\backslash\backslash = -w\left(\frac{L}{2}\right)$

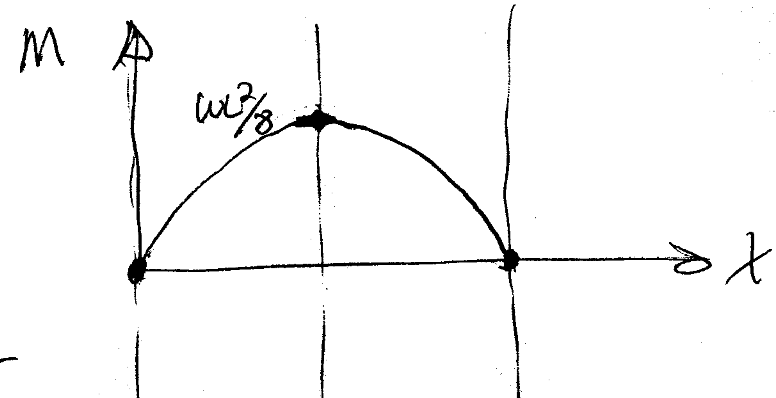
$///// = -w\left(\frac{L}{2}\right)$

$V_A = \frac{wL}{2}$

$V_B = V_A -$ area $\backslash\backslash\backslash\backslash$

$= \frac{wL}{2} - \frac{wL}{2} = 0$

$V_C = V_B -$ area $////$ $= 0 - \frac{wL}{2}$



① $= \frac{1}{2}\left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = \frac{wL^2}{8}$

② $= -\frac{1}{2}\left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = \frac{-wL^2}{8}$

$\overline{\quad 0 \quad}$ ✓

$M_A = M_C = 0$

$M_C = M_A +$ ① $+$ ② $= 0$ ✓

$M_B = M_A +$ ① $= 0 + \frac{wL^2}{8}$

$M_C = M_B +$ ② $= \frac{wL^2}{8} - \frac{wL^2}{8} = 0$

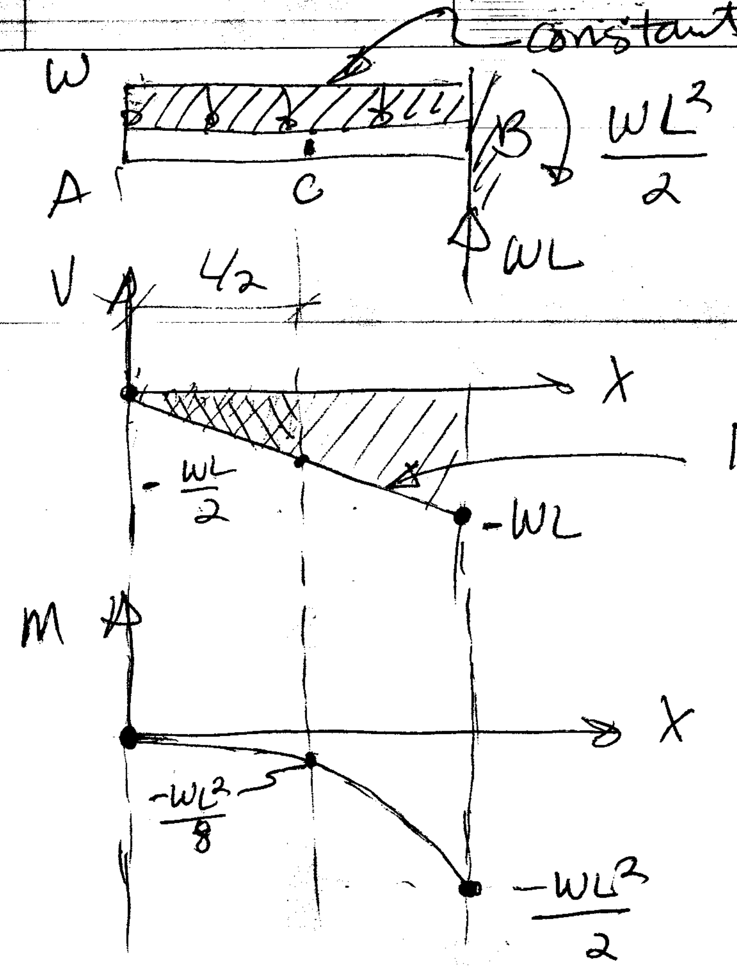
Since V is linear, M is parabola

At B, $V=0$

$\frac{dM}{dx} = V = 0$ so

slope of M curve is zero @ B (maximum)

Example:



$V_A = 0$

$V_B = V_A - \text{area} ////$

$= -wL$

$M_A = 0$

$M_B = M_A + ////$

$= 0 - \frac{1}{2}(wL)(L)$

$= -\frac{wL^2}{2}$

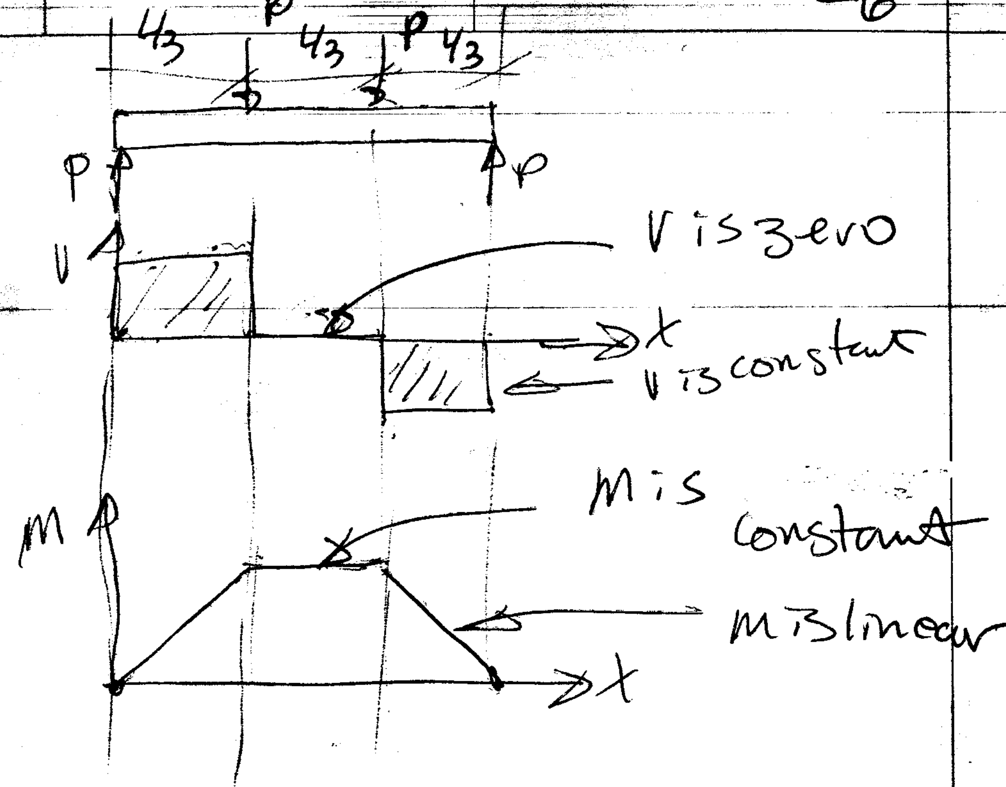
parabola, but which way?

Notice: $V \neq 0$ @ B, so slope of M curve $\neq 0$ @ B

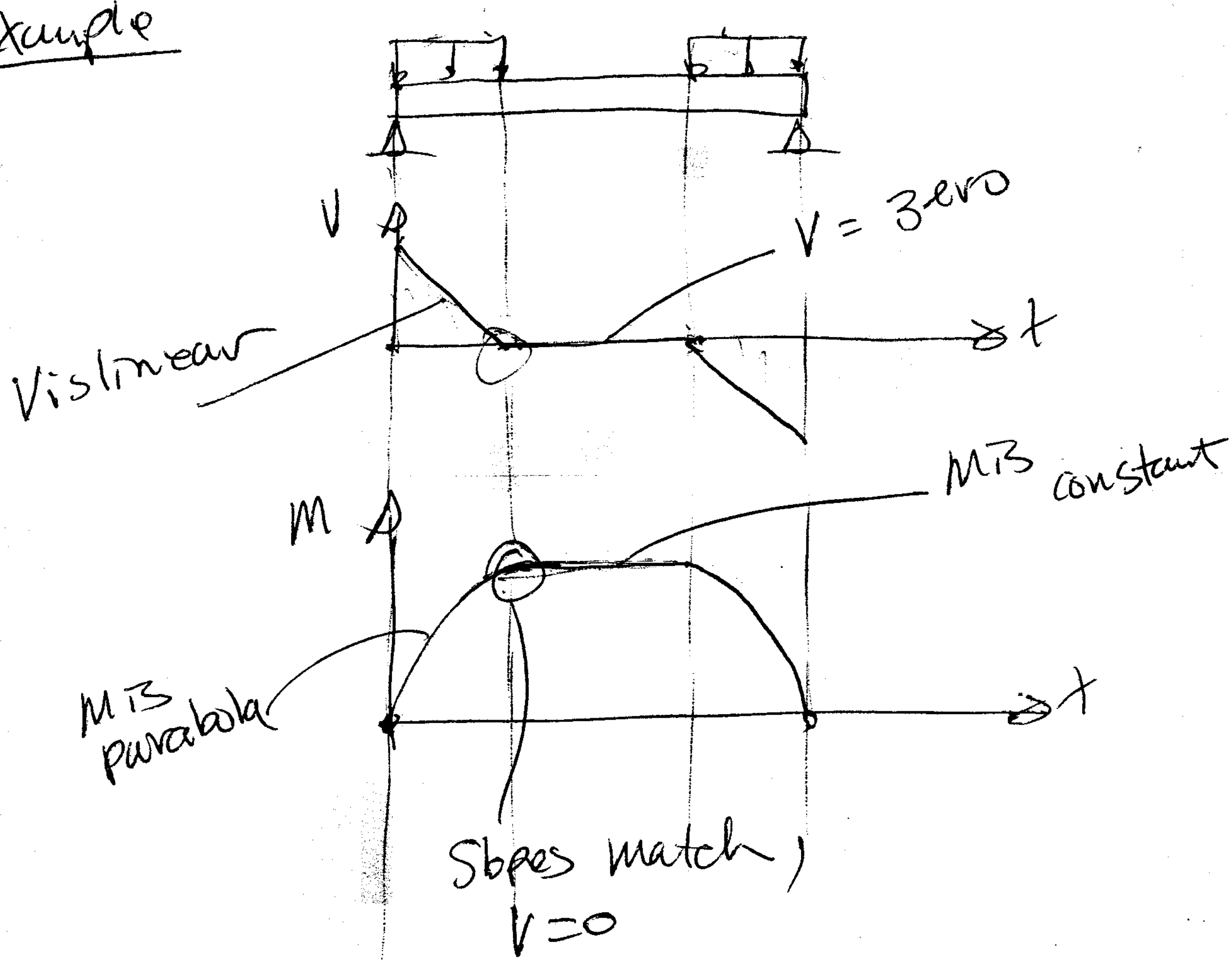
→ check ½ way point, C

$M_c = M_A + \text{XXXX} = 0 - \frac{1}{2}\left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = -\frac{wL^2}{8}$

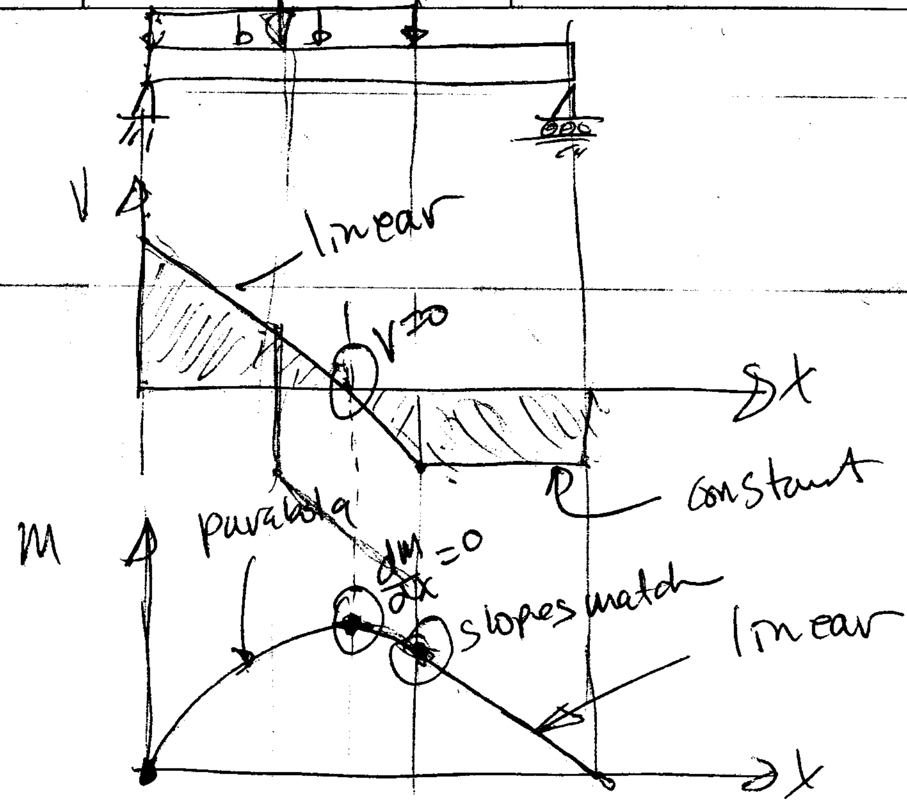
Example:



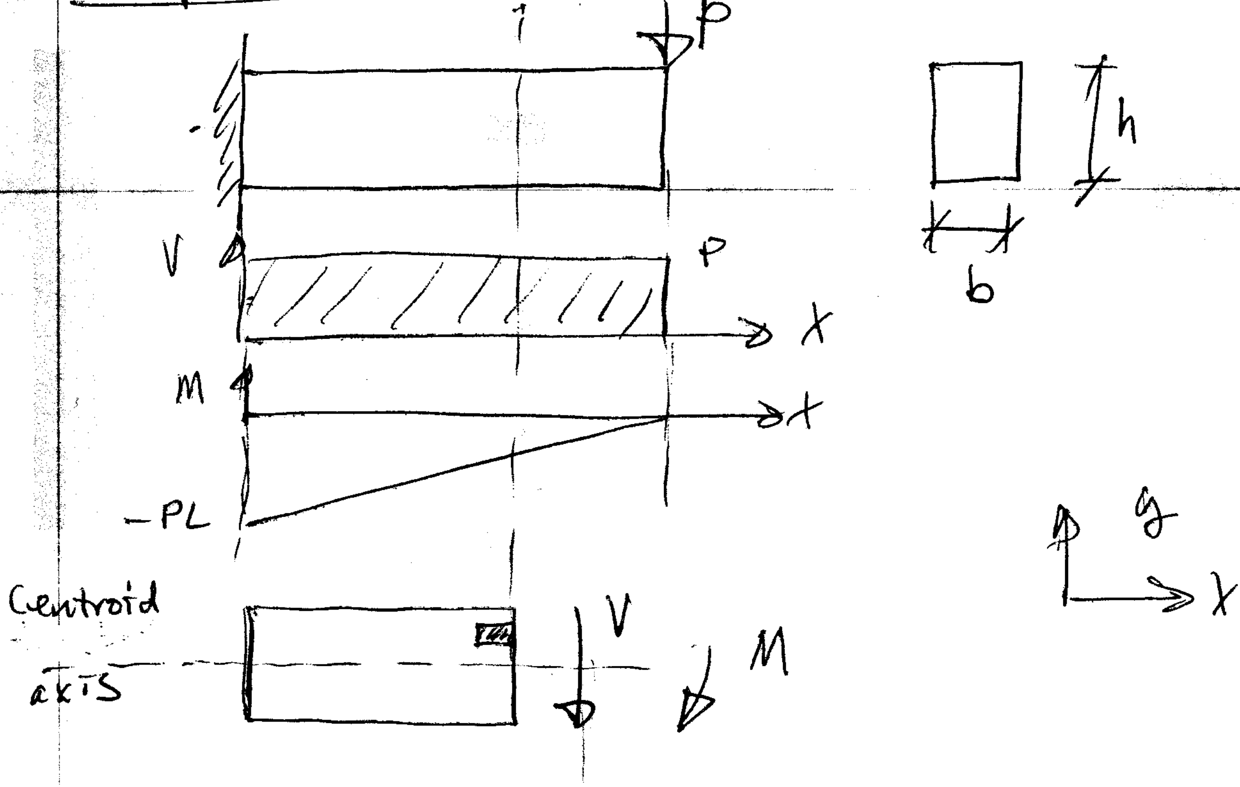
Example



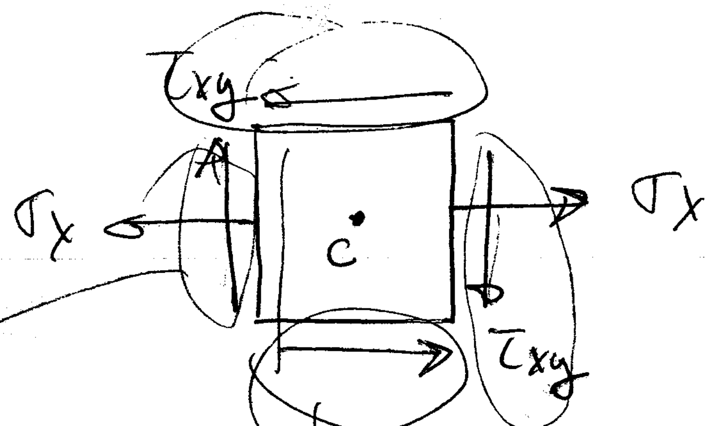
Example


b
V
linear
V=0
x
constant
M
parabola
$\frac{dM}{dx}=0$
slopes match
linear
x

Chapter 6 - Shear Stresses in beams



Shear stress due to V
Normal stress due to M



Note: $\tau_{xy} \neq \frac{V}{A}$!!

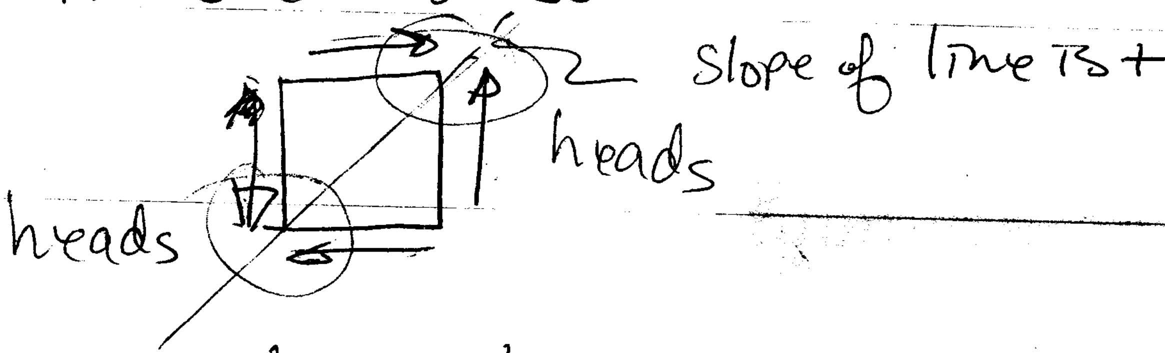
Shear stresses always come in 2 pairs!!

$\Sigma F_y = 0$

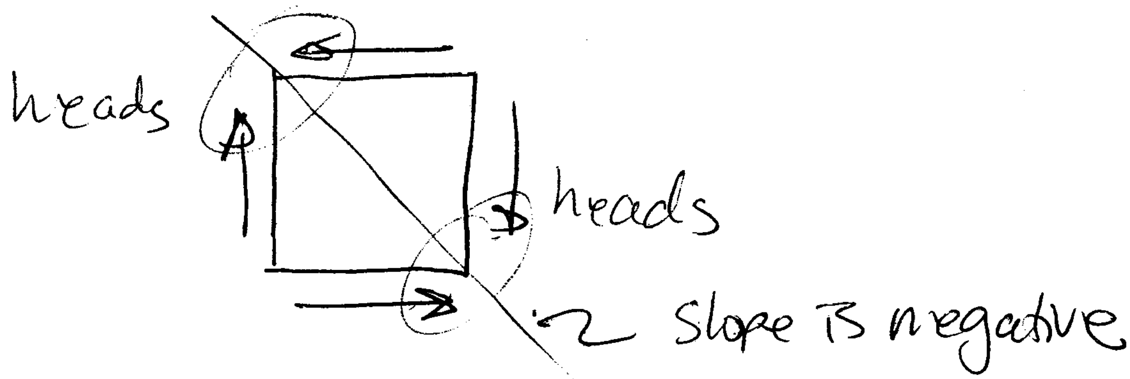
$\Sigma M_c = 0$

and arrowheads point @ eachother @ corners!

Positive shear stress:



negative shear stress:



$\tau = \frac{VQ}{It}$, V = Vertical shear force

I = moment of inertia for entire section

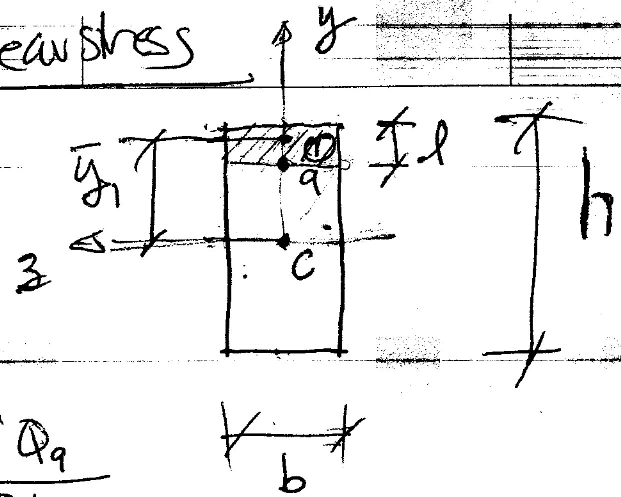
t = thickness of "cut face"

Q = $A\bar{y}$ for section that would fall off ~~at~~ due to "cut"

$q = \frac{VQ}{I}$ = shear force / unit length

Also: $\tau = \frac{q}{t}$

Example:



$$\tau_a = ? = \frac{V Q_a}{I t_a}$$

$$I = \frac{1}{12} b h^3$$

$Q_a = A_1 \bar{y}_1$ $\quad\bar{y}_1$ = distance from section centroid to ① centroid

$A_1 = bl$ = area of piece that would fall off due to cut

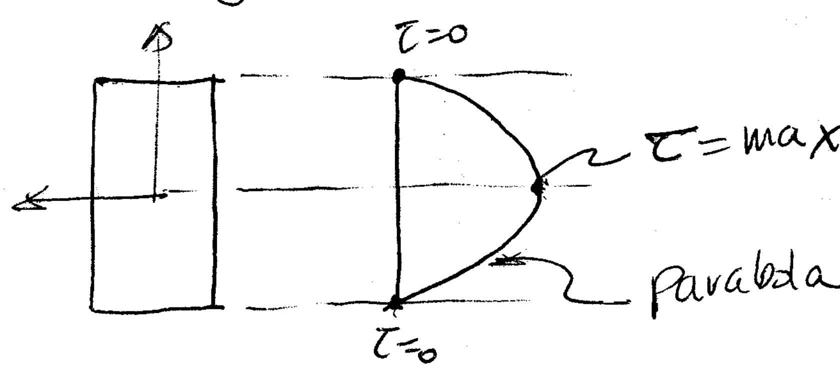
t = thickness of cut $= b$

Notice: for top face, bottom face, $Q = 0$ since $A = 0$

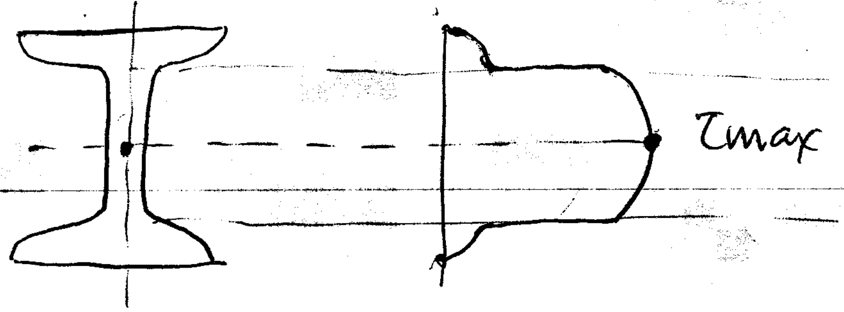
∴ At centroid, c $\quad Q = \max$ so $\tau = \max$

horizontal cut ⟹ horizontal shear stress

Distribution:

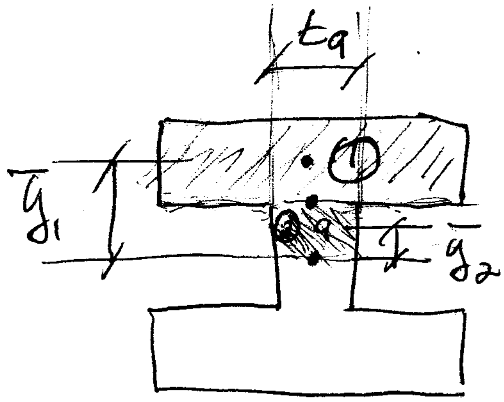


Similar for other sections:



So: $\tau = max$ @ centroid axis and $\tau = 0$ on top, bottom faces.

Another Example



$$\tau_a = \frac{V Q_a}{I t_a}$$

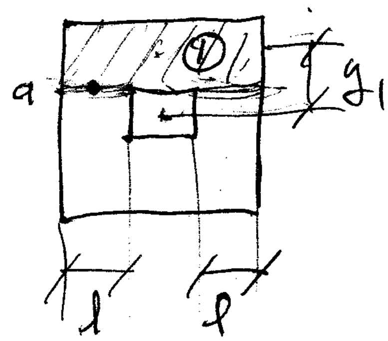
$$Q_a = Q_1 = A_1 \bar{y}_1$$

$$\tau_{max} = \frac{V Q_{max}}{I t}$$

$$Q_{max} = Q_1 + Q_2$$
$$t = t_a$$

ex

$$\tau_a = \frac{V Q_a}{I t_a}$$

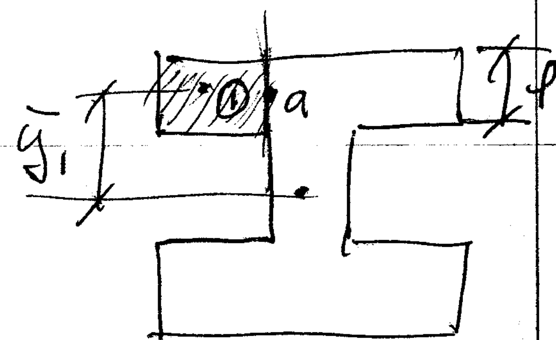


$$Q_a = Q_1 = A_1 \bar{y}_1$$

$t_a = l + l$ (2 cut faces!)

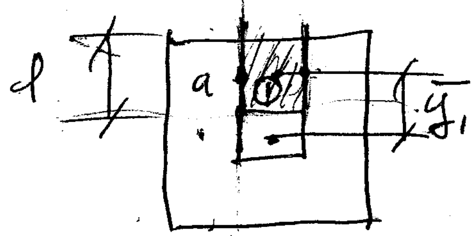
Vertical shear stress

Same formula, but make a vertical cut



$$\tau_a = \frac{VQ_a}{It}$$

$Q_a = Q_1 = A_1 \bar{y}_1$, $t = l$



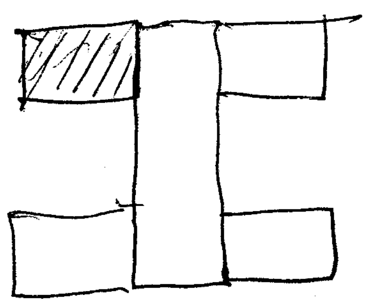
$$\tau_a = \frac{VQ_a}{It_a}$$

$Q_a = A_1 \bar{y}_1$

$t_a = l + l$ (2 cut faces)

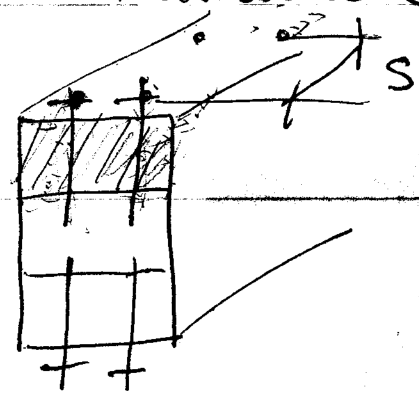
need 2 cuts to separate off a piece that could fall off

Sometimes compare to shear allowable stress for glued joint



Sometimes check — allowable load

nailed connection



first, find $q = \frac{VQ}{I}$

q = force / unit length

horizontal shear force $= qs = F_s$

two nails: $F_{all} = \frac{F_s}{2} = \frac{qs}{2}$

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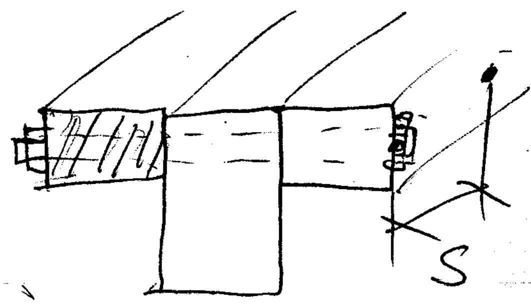
or Bolted connection:



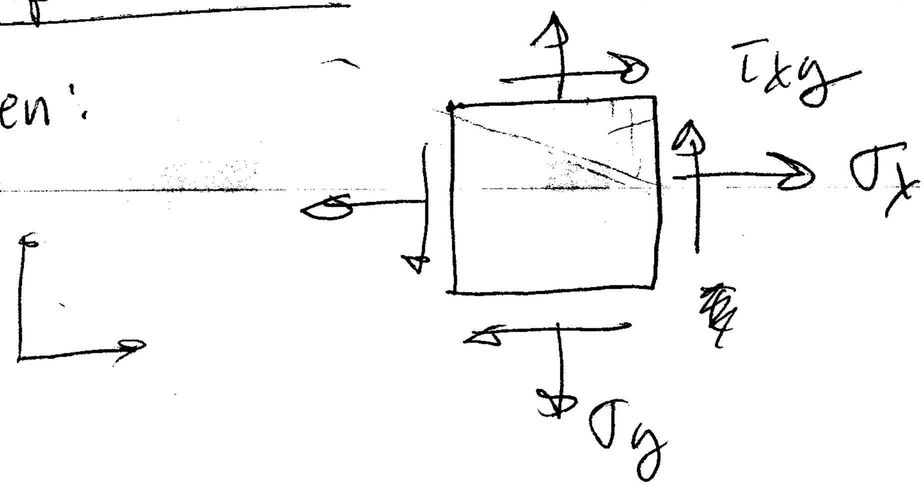
$q = \frac{VQ}{I}$

Shear force on each bolt $= qs = F_s$

bolt shear stress $= \frac{F_s}{A_{bolt}} = \frac{F_s}{\pi r^2}$

Chapter 7 - Stress Transformation

Given:



find: principal stresses, σ_{max}, σ_{min}
principal planes, θ_p
Max In-plane shear stress, τ_{max}
Plane of max shear stress, θ_s

See book for plug + chug formulas
(p. 428 - 430)

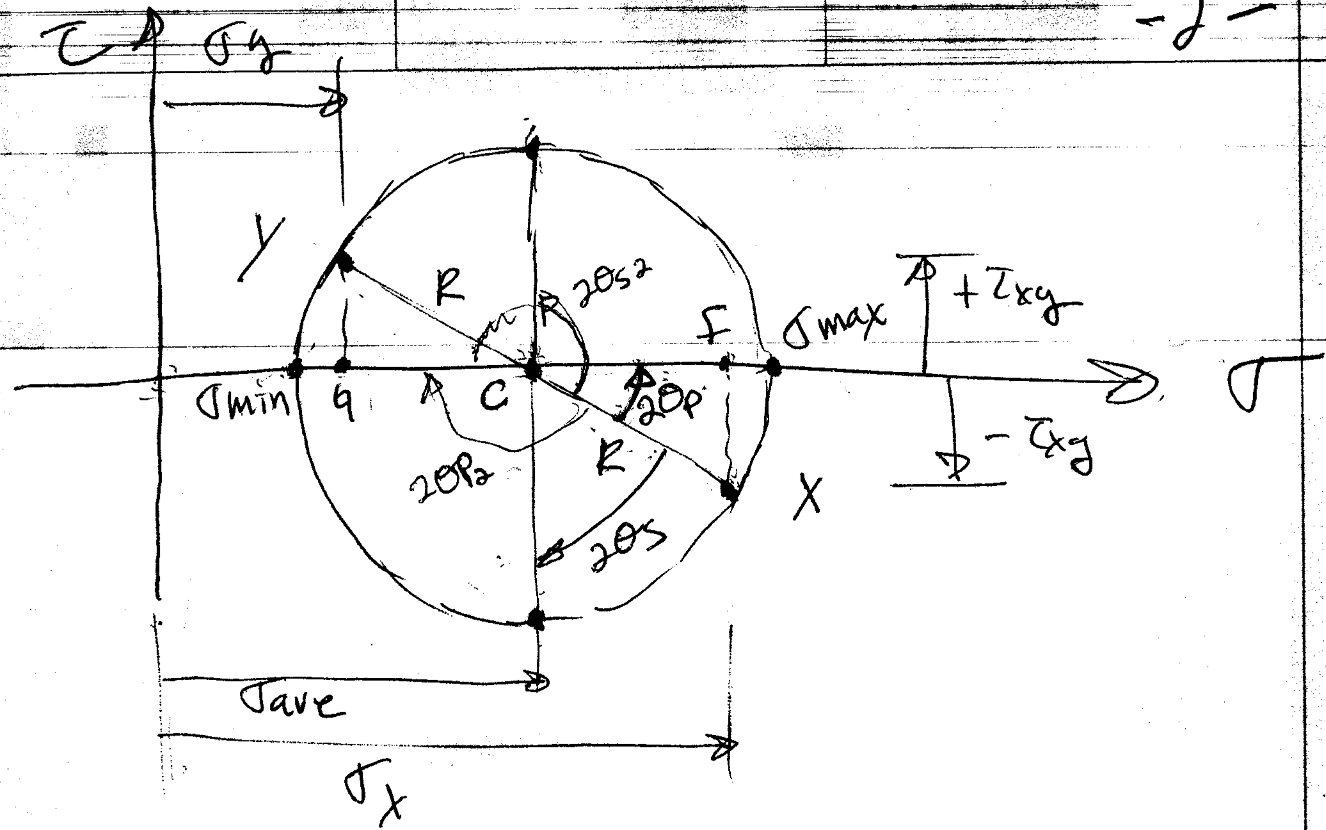
Mohr's circle

plot two points:

X: $(\sigma_x, -\tau_{xy})$ say $\sigma_x > \sigma_y$

Y: $(\sigma_y, +\tau_{xy})$

Center of circle $= \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$



Draw circle centered @ σ_{ave}, C + passing through X, Y

$R = \sqrt{(CF)^2 + (XF)^2}$ here, $CF = \sigma_x - \sigma_{ave}$

$XF = \tau_{xy}$

~~After~~

$\sigma_{max} = \sigma_{ave} + R$

$\sigma_{min} = \sigma_{ave} - R$

$\tau_{max} = R$

$\tan 2\theta_p = \frac{XF}{CF} \rightarrow \theta_p = \#$, here, CCW

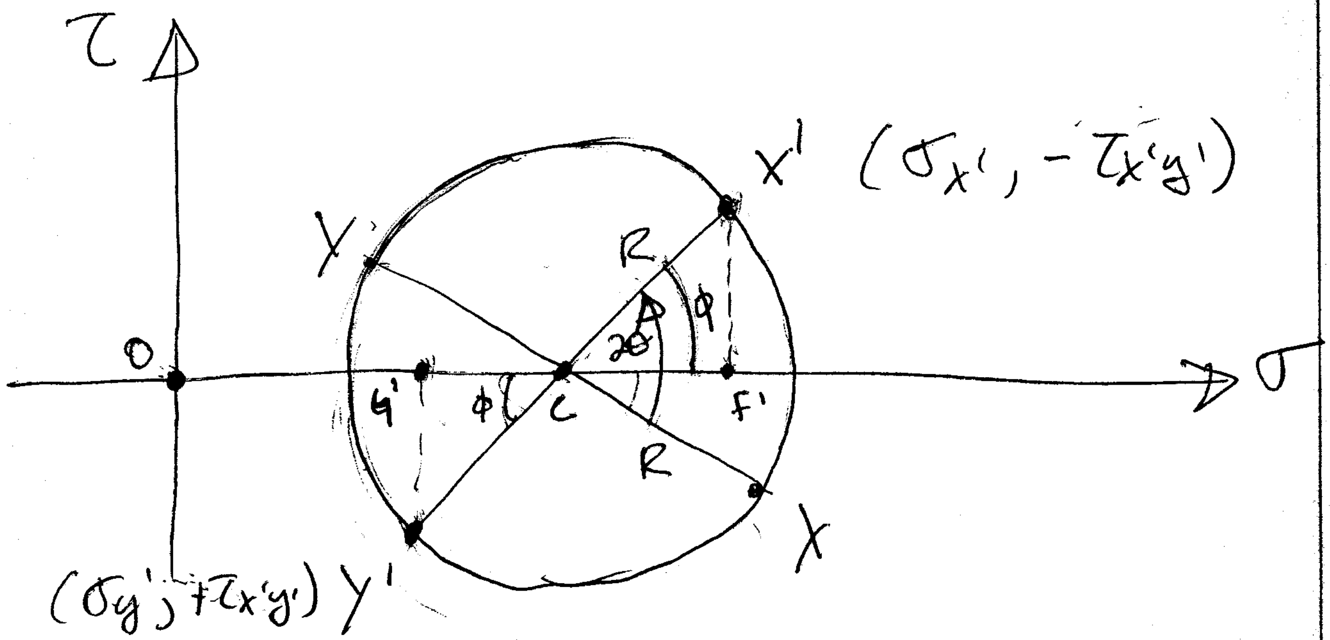
(Also $2\theta_{p2} = 180 - 2\theta_p$) (CW)

$2\theta_s = 90 - 2\theta_p$, here θ_s is CW

(also $\theta_{s2} = 180 - 2\theta_s$) (CCW)

Determine $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ If element is rotated same angle θ

Ex: θ, ccw (where $2\theta > 2\theta_p$)



$\phi = 2\theta - 2\theta_p$

$\sigma_x' = OC + CF'$

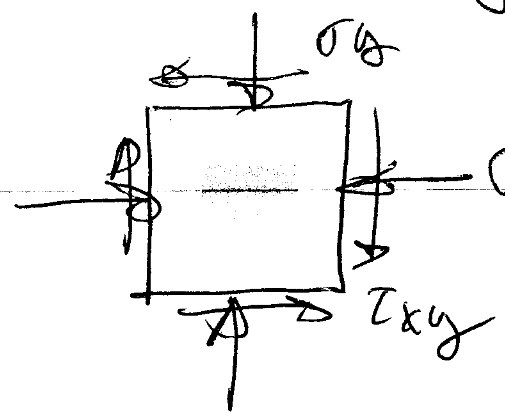
$= \sigma_{ave} + R\cos\phi$

$\sigma_y' = OC - CG' = \sigma_{ave} - R\cos\phi$

$\tau_{x'y'} = - R\sin\phi < 0$

because X' is $-\tau_{x'y'}$ and $\tau_{x'y'} > 0$ on plot

What about all negative?

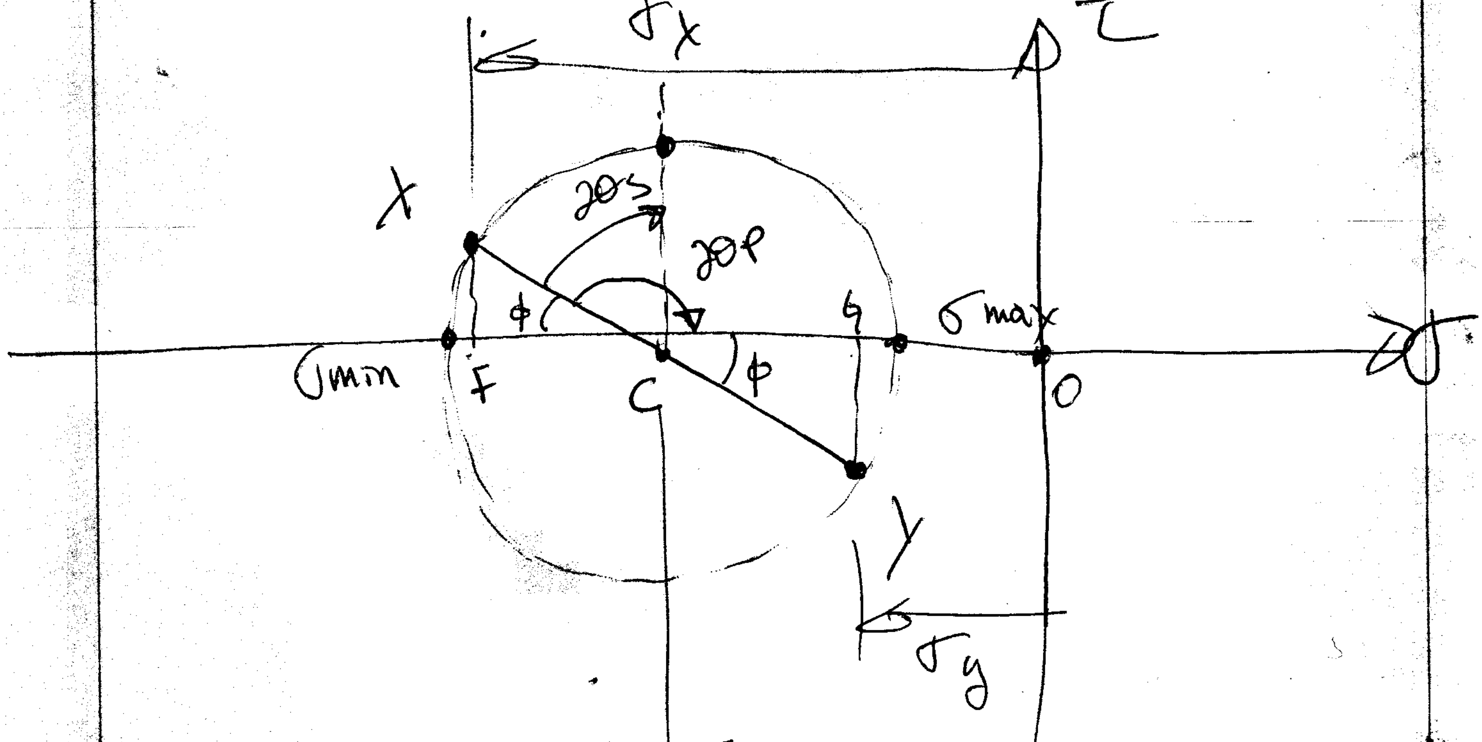


$$\left.\begin{array}{l}\sigma_x < 0 \\ \sigma_y < 0\end{array}\right\} \text{ say } |\sigma_x| > |\sigma_y|$$

$\tau_{xy} < 0$

$$X : (\sigma_x, -\tau_{xy}) = (-\#, +\#)$$

$$Y : (\sigma_y, +\tau_{xy}) = (-\#, -\#)$$



$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} < 0$$

$$R = \sqrt{(C_G)^2 + (C_Y)^2} \quad , \quad C_G = |\sigma_{ave}| - |\sigma_y|$$

$$C_Y = |\tau_{xy}|$$

$$\sigma_{min} = \underbrace{\sigma_{ave}}_{<0} - R \quad < 0$$

$$\sigma_{max} = \underbrace{\sigma_{ave}}_{<0} + R \quad < 0$$

$$\tau_{max} = R$$

$$\cancel{\theta_p} \quad \phi = \tan^{-1}\left(\frac{C_Y}{C_G}\right)$$

$$2\theta_p = 180 - \phi \quad , \theta_p \text{ IS } \underline{\underline{CW}}$$

$$2\theta_s = 90 - \phi \quad , \theta_s \text{ IS } CW$$

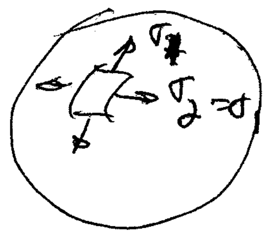
Pressure vessels:

1) spherical:

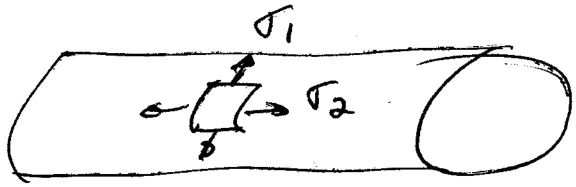
$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

p = pressure
r = inner radius
t = thickness

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{pr}{4t}$$



2) cylindrical



$$\sigma_1 = \text{hoop stress} = \frac{pr}{t}$$

$$\sigma_2 = \text{longitudinal stress} = \frac{pr}{2t} = \frac{\sigma_1}{2}$$

$$\tau_{max} = \sigma_2 = \frac{pr}{2t}$$