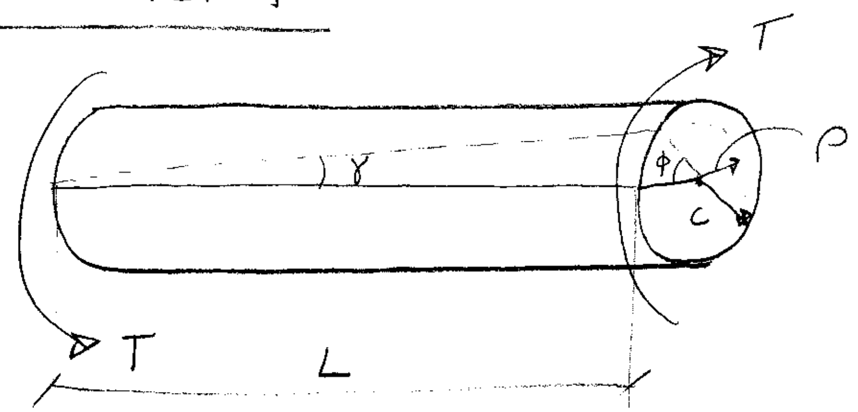
Chpt 3 - Torsion

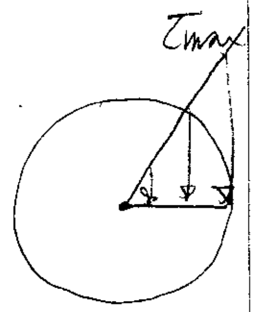


T = torque
c = radius
ρ = radial distance

γ = shear strain = $\frac{\rho\phi}{L}$, $\gamma_{max} = \frac{c\phi}{L}$

ϕ = angle of twist (radians) = $\frac{TL}{JG}$

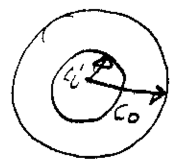
τ = shear stress = $\frac{T\rho}{J}$, $\tau_{max} = \frac{Tc}{J}$



J = polar moment of inertia

solid cylinder: $J = \frac{\pi}{2} c^4$

hollow cylinder: $J = \frac{\pi}{2}(c_o^4 - c_i^4)$



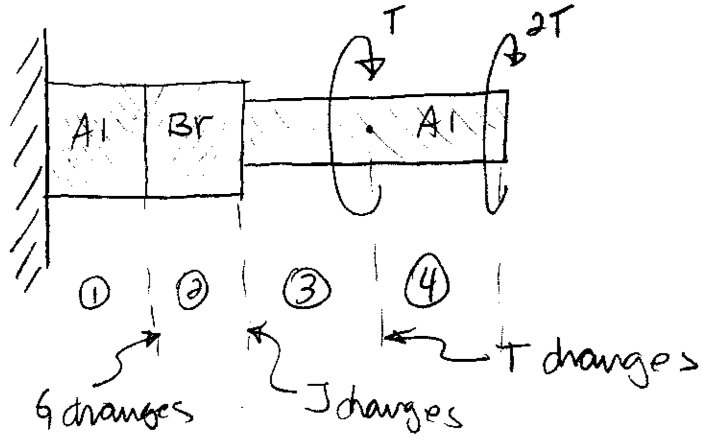
$\tau = \gamma G$, G = shear modulus

Rods in Series:

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

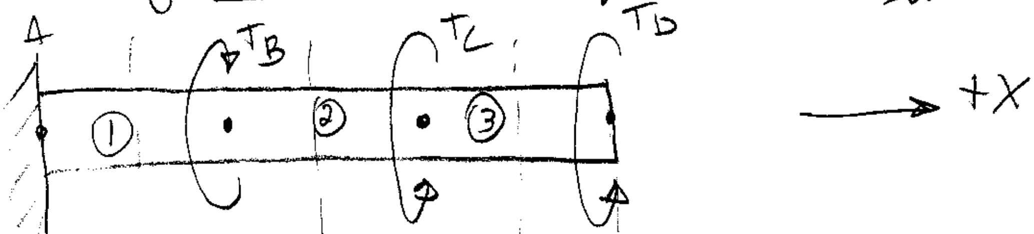
Divide into portions - each time T, J, G change

Ex:

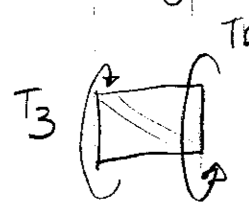


$$\phi = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} + \dots \text{ etc.}$$

Determining Internal torques - Cut open + Sum Moments

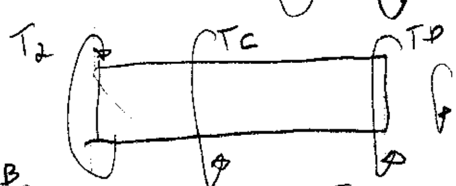


Use these internal torques in $\phi = \frac{TL}{JG}$, $\tau = \frac{T\rho}{J}$ etc.



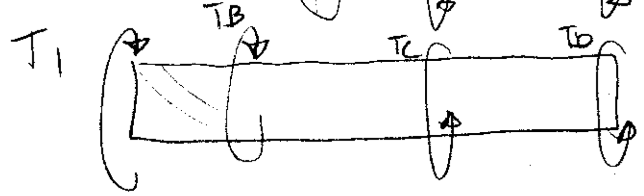
$$\sum M_x = T_D - T_3 = 0$$

$$T_3 = T_D$$



$$\sum M_x = T_D + T_C - T_2 = 0$$

$$T_2 = T_D + T_C$$

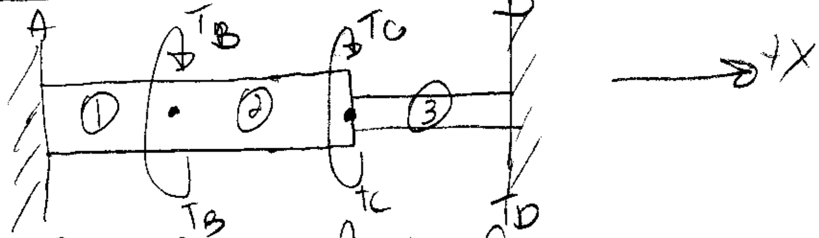


$$\sum M_x = T_D + T_C - T_B - T_1 = 0$$

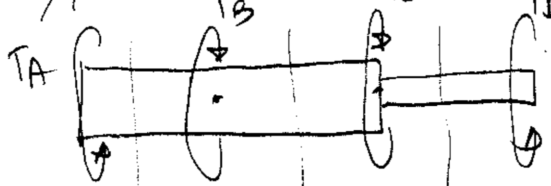
$$T_1 = T_D + T_C - T_B$$

Statically indeterminant rods

- Series:



1) statics



$\Sigma M_x = 0$

$\rightarrow \boxed{T_A + T_D = T_B + T_C}$

$\left.\begin{array}{l}\rightarrow T_1 \\ \rightarrow T_2 \\ \rightarrow T_3\end{array}\right\}$ in terms of T_A, T_D

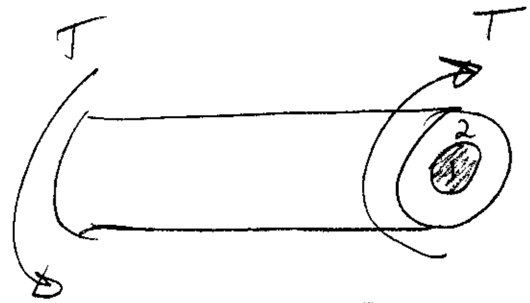
2) deformation:

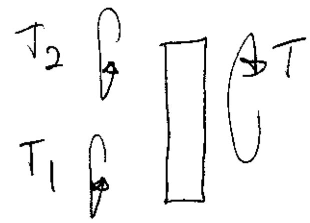
$$\phi_1 + \phi_2 + \phi_3 = 0$$

$$\boxed{\frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} + \frac{T_3 L_3}{J_3 G_3} = 0}$$

2 eqns, 2 unknowns

Parallel





statics: $\boxed{T = T_1 + T_2}$

Deformations:

$$\phi_1 = \phi_2 \rightarrow \boxed{\frac{T_1 L_1}{J_1 G_1} = \frac{T_2 L_2}{J_2 G_2}}$$

$L_1 = L_2$

2 eqns, 2 unknowns

once you know Torques from above
- can Find ϕ, γ, τ, etc.

Design of transmission shafts

P = Power — N·m/s = watts ; hp = 550 ft·lb/s

f = freq. of rotation — Hz ; rpm = $\frac{1}{60}$ Hz

$$P = 2\pi T f$$

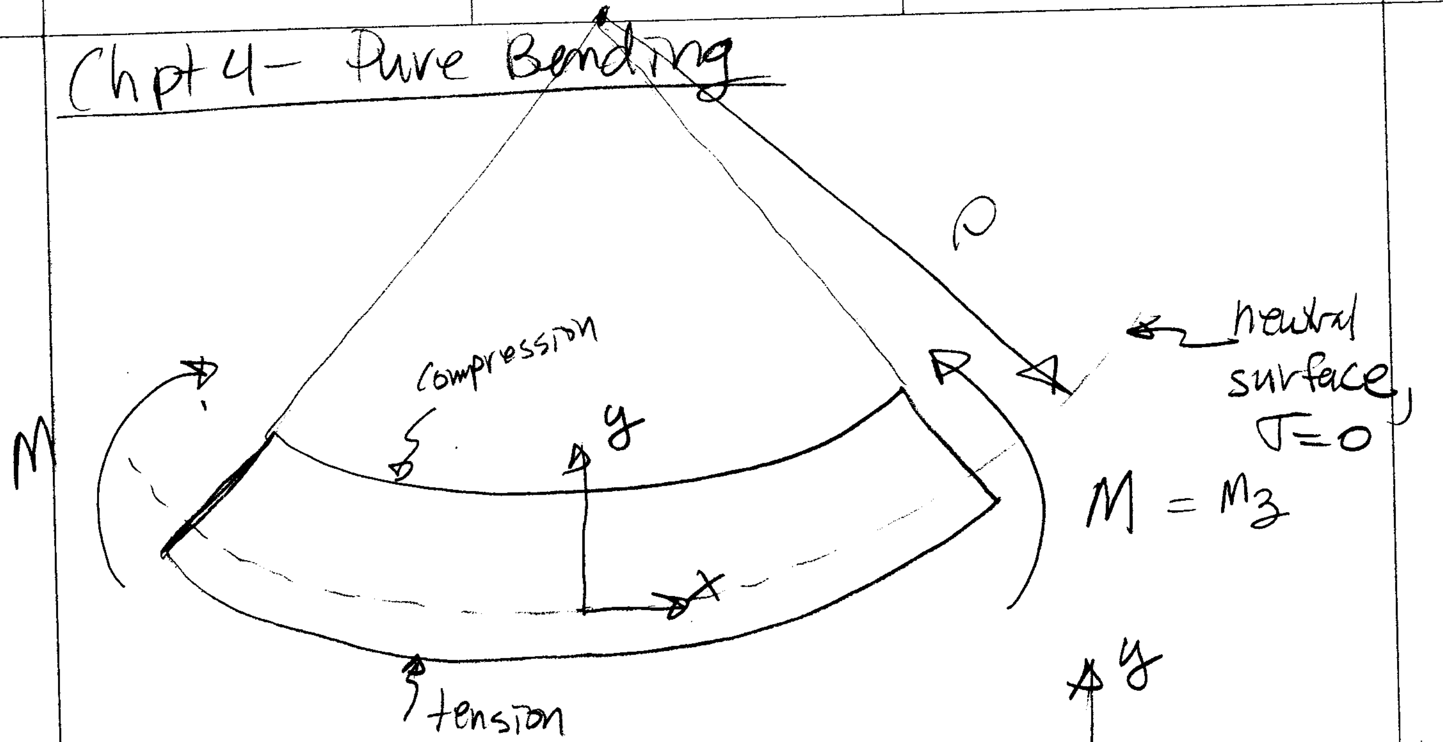
$$T = \frac{P}{2\pi f}$$

given P, f - find T

$$\tau_{all} = \tau_{max} = \frac{Tc}{J}, \quad \frac{J}{c} = \frac{T}{\tau_{all}}$$

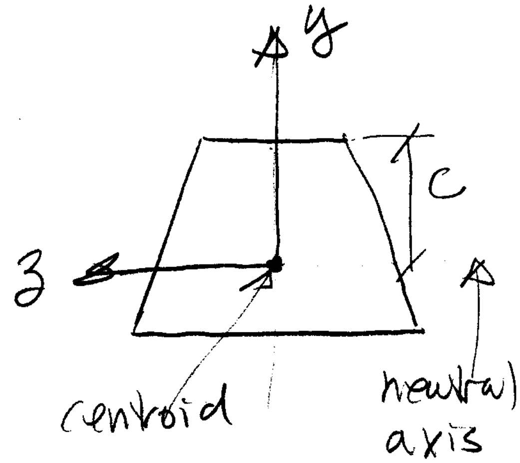
determine req'd shaft cross section to satisfy req'd $\frac{J}{c}$

Chpt 4 - Pure Bending



$M = M_z$

$$\sigma_x = \sigma = -\frac{My}{I}, \quad I = I_z$$

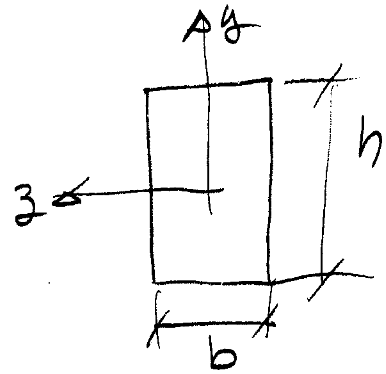


$$\sigma_{max} = \frac{Mc}{I}$$

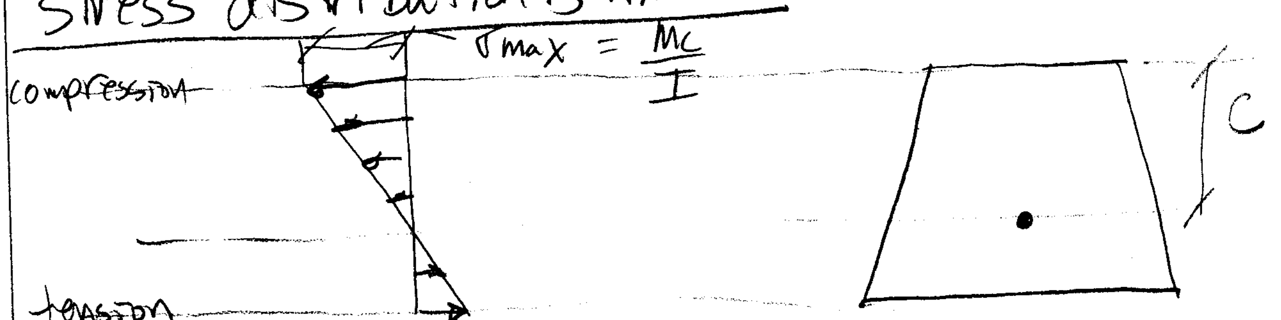
c = maximum y distance from neutral axis

I = moment of inertia, ex:

$$I = I_z = \frac{1}{12} b h^3$$



Stress distribution is linear



ρ = radius of curvature

$\frac{1}{\rho} = \frac{M}{EI}$, E = young's modulus

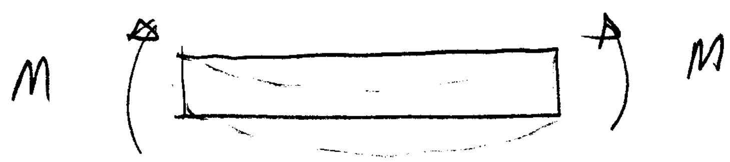
ρ = large, beam is "flatter"
ρ = small, beam is "more curved"

$\sigma_{max} = \frac{Mc}{I} = \frac{M}{S}$

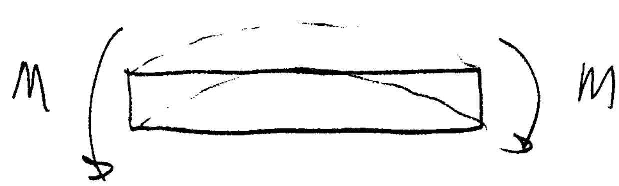
S = section modulus $= \frac{I}{c}$

$\varepsilon_x = \frac{\sigma_x}{E} = -\frac{My}{EI}$ axial strain

$\varepsilon_y = \varepsilon_z = -\nu \varepsilon_x$ lateral strain

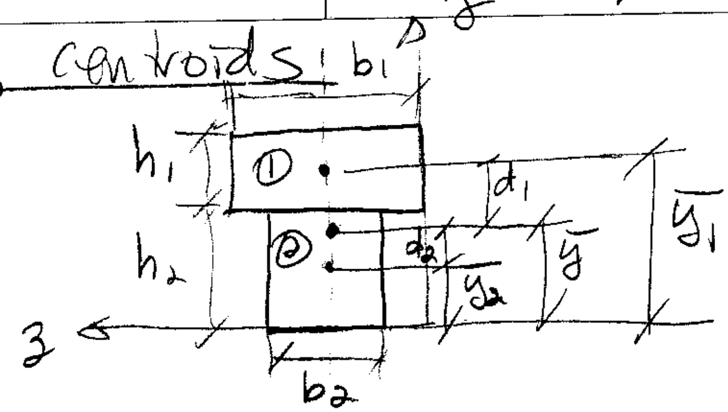


positive moment: beam smiles
top in compression
bottom in tension



negative moment: beam frowns
top is in tension
bottom - compression

Determining I, centroids:



$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

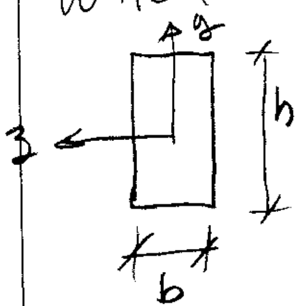
$$= \frac{b_1 h_1 y_1 + b_2 h_2 y_2}{b_1 h_1 + b_2 h_2}$$

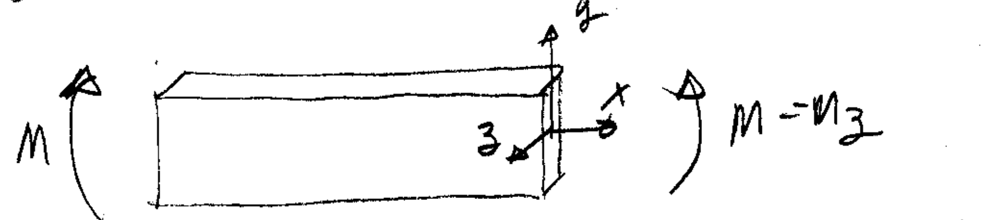
$$I_z = I_{1z} + I_{2z}$$

$$I_{1z} = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 \quad , \quad d_1 = |\bar{y}_1 - \bar{y}|$$

$$I_{2z} = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 \quad , \quad d_2 = |\bar{y}_2 - \bar{y}|$$

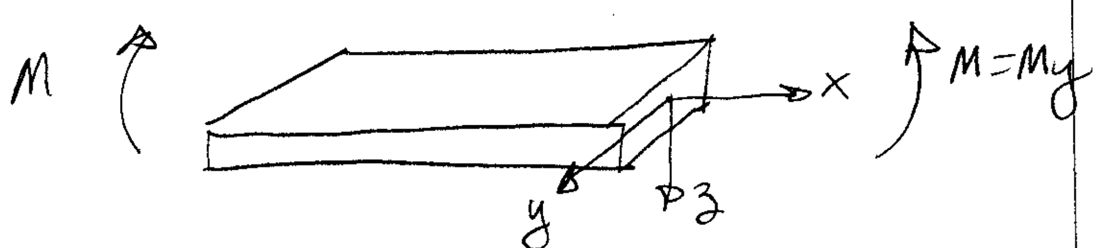
Which I to use?





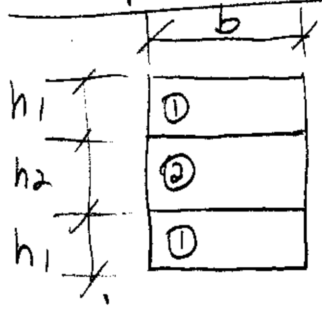
use $I_z = \frac{1}{12} b h^3$ (h > b, stiffer)

recall 2x4 demo



use $I_y = \frac{1}{12} h b^3$ (b < h, weaker)

Composite Sections



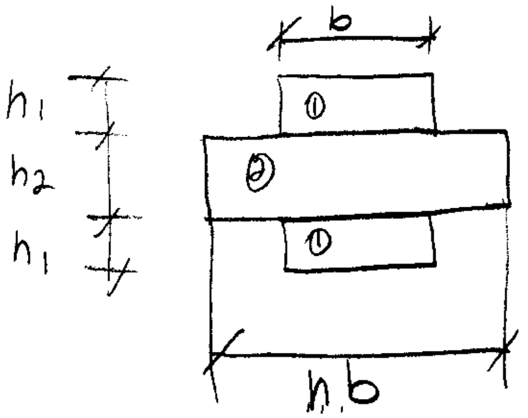
Say $E_2 > E_1$

$$n = \frac{E_{stiffer}}{E_{weaker}} = \frac{E_2}{E_1} > 1$$

(1) Weaker material: $\sigma = -\dfrac{My}{I}$

(2) Stiffer material: $\sigma = -n\dfrac{My}{I}$

Determine I: Create transformed section

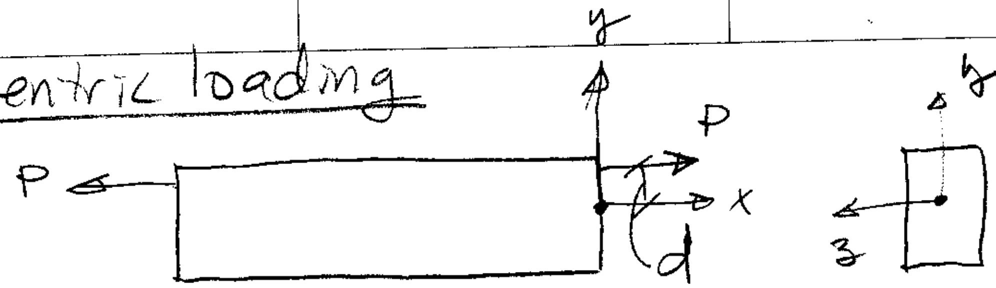


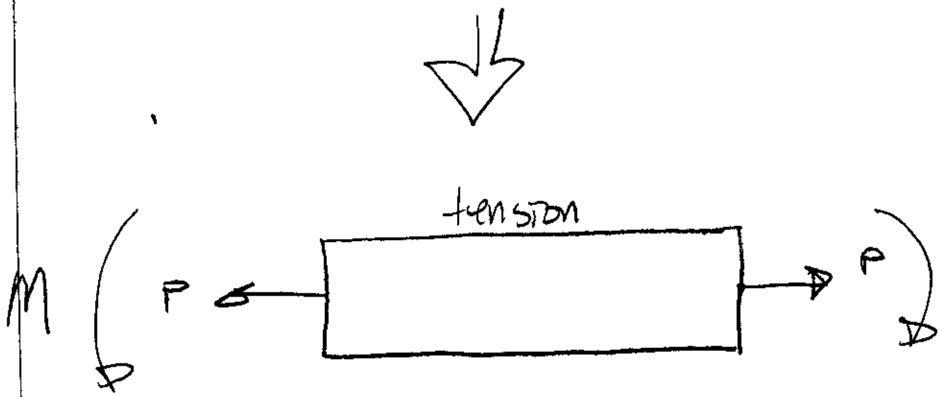
$$I_1 = \frac{1}{12} b h_1^3$$

$$I_2 = \frac{1}{12} (nb) h_2^3$$

$$I = 2\left[I_1 + A_1 d_1^2\right] + I_2$$

Eccentric loading





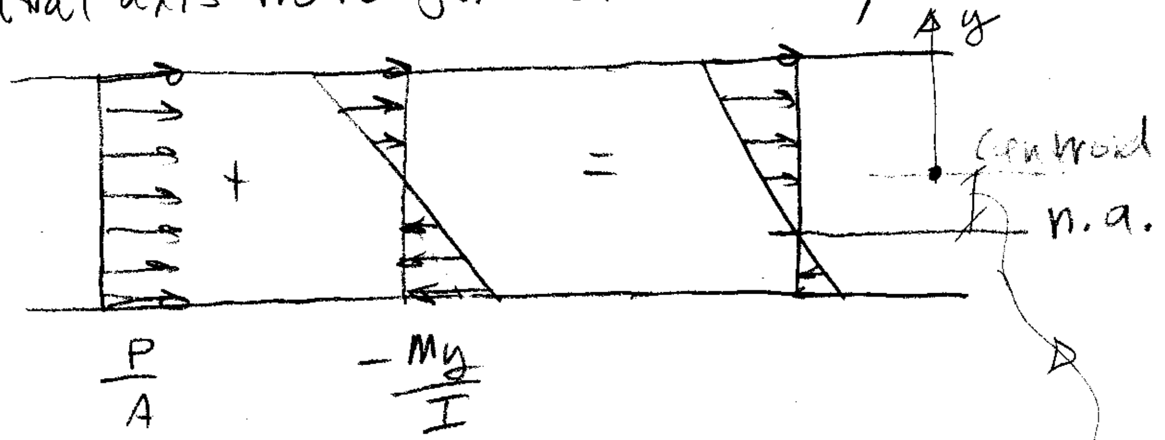
$M = Pd$

$$\sigma = \frac{P}{A} - \frac{My}{I}$$

Note: $P > 0$ tension
$M < 0$ negative bending

$$= \frac{P}{A} - \frac{(-Pd)\,y}{I}$$ → tension on top face

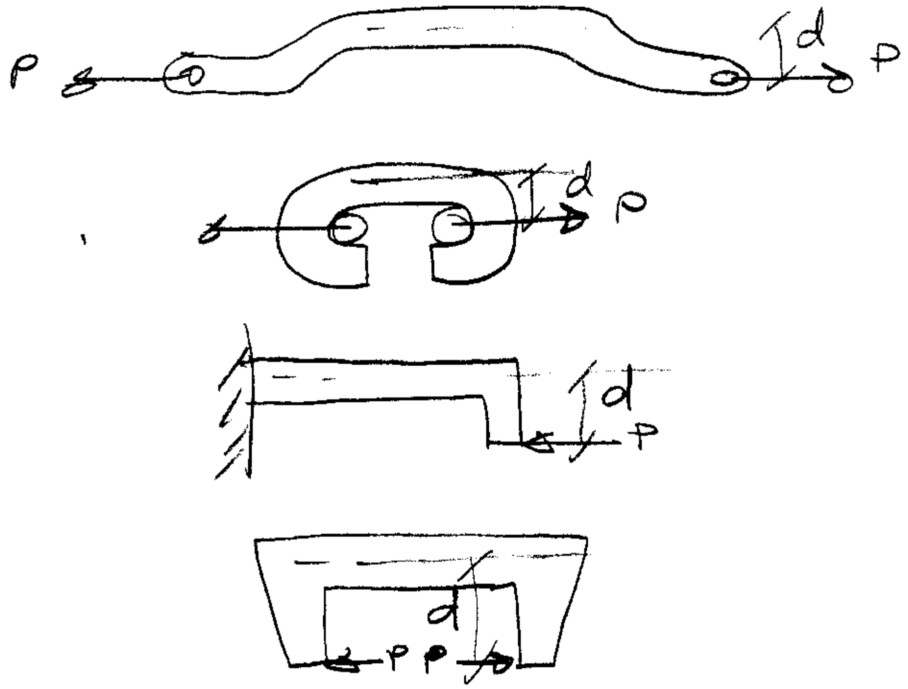
Neutral axis no longer coincides w/ centroid:



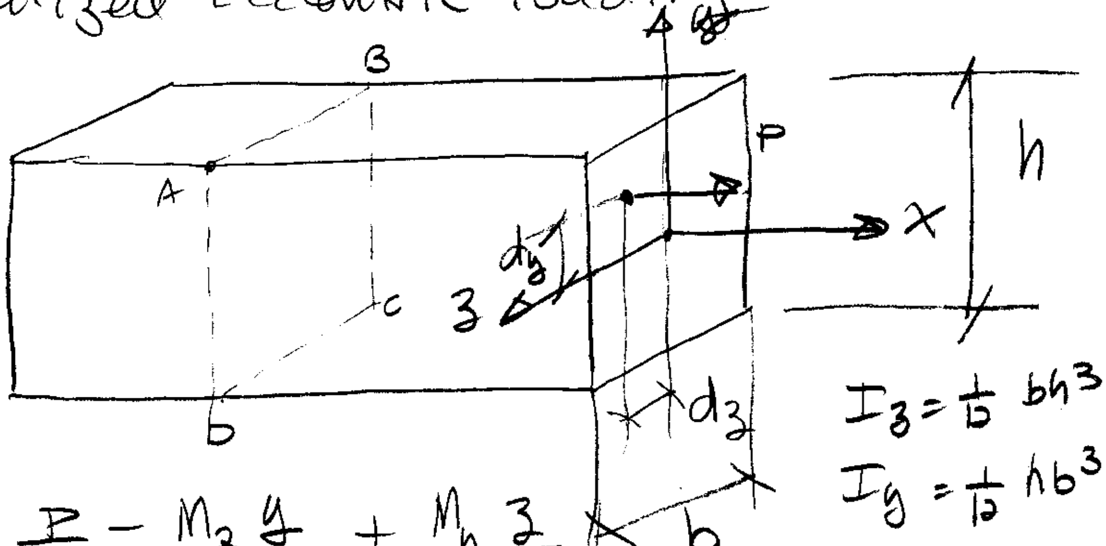
Neutral axis: $\sigma = \frac{P}{A} - \frac{My}{I} = 0$

$$y = \frac{PI}{AM} = \frac{PI}{A(-Pd)} = -\frac{I}{Ad}$$

other ways to get eccentric loading:



Generalized Eccentric loading



$$I_z = \frac{1}{12} bh^3$$

$$I_y = \frac{1}{12} hb^3$$

$$\sigma = \frac{P}{A} - \frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

by inspection: $M_z = Pd_y$, negative

$M_y = Pd_z$, positive

(positive M_z = compression on top)
(positive M_y = tension on front)

$\hat{i}\ \hat{j}\ \hat{k}\ \hat{i}\ \hat{j}$ $\xrightarrow{\quad} +$

$- \xleftarrow{\quad}$

By Vectors;

$$\vec{P} = P\hat{i}, \quad \vec{d_y} = +d_y\hat{j}, \quad \vec{d_z} = +d_z\hat{k}$$

$$\vec{M_z} = \vec{d_y} \times \vec{P} = (d_y\hat{j}) \times (P\hat{i}) = -d_yP\hat{k}$$

$$\vec{M_y} = \vec{d_z} \times \vec{P} = (d_z\hat{k}) \times (P\hat{i}) = +d_zP\hat{j}$$

$$\sigma = \underbrace{\frac{P}{A}}_{\sigma_1} \underbrace{- \frac{(-d_yP)y}{I_z}}_{\sigma_2} + \underbrace{\frac{(d_zP)z}{I_y}}_{\sigma_3}$$

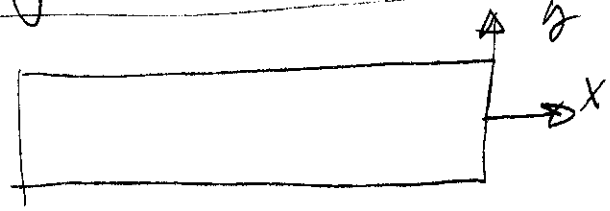
Maximum tension; by inspection: A

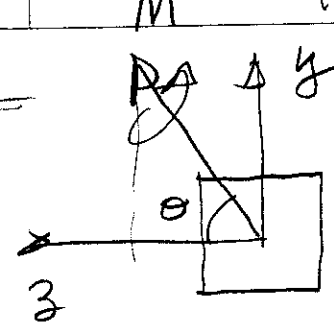
by math: $\sigma_1 > 0$
$\sigma_2 > 0$ if $y > 0$, $y_{max} = \frac{h}{2}$
$\sigma_3 > 0$ if $z > 0$, $z_{max} = \frac{b}{2}$
} Point A

Max Compression / Min. Tension

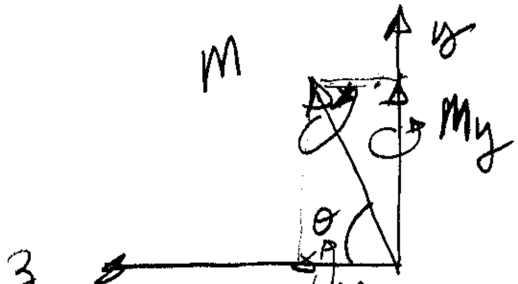
$\sigma_1 > 0$
$\sigma_2 < 0$ when $y < 0$; $y = -h/2$
$\sigma_3 < 0$ when $z < 0$; $z = -b/2$
} Point C

Unsymmetric bending





Break vector $\vec{M}$ into y, z components

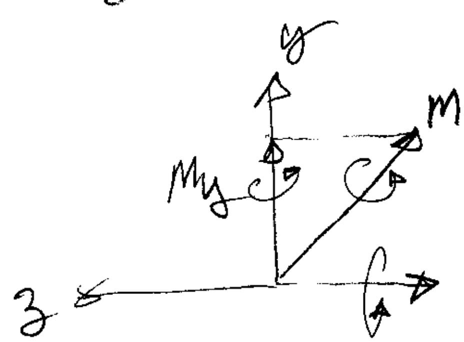


$M_y = M\sin\theta$

$M_z = M\cos\theta$

here both are +

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



$M_z \rightarrow$ negative