Final Exam (last 3wks) Review session

9 - Beam Deflections

ρ = radius of curvature

$$\frac{1}{\rho} = \frac{M(x)}{EI} \approx \frac{d^2y}{dx^2}$$

Integrate once:

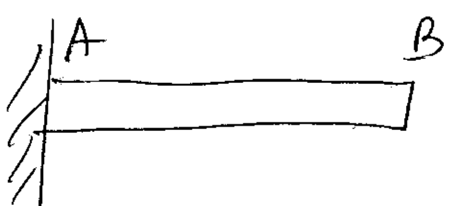
slope (in radians)

$$\frac{dy}{dx} = \tan\theta \approx \theta(x) \rightarrow \theta(x) = \int \frac{M(x)}{EI}\,dx + C_1$$

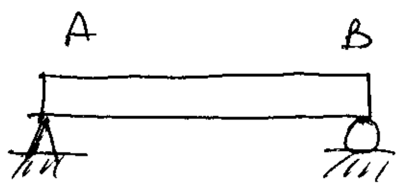
Integrate again:

$$y(x) = \int \theta(x)\,dx + C_2$$ = deflection = elastic curve

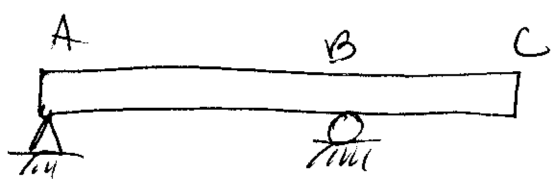
Apply Boundary Conditions to evaluate C_1, C_2:



$y_A = 0$, $\theta_A = 0$



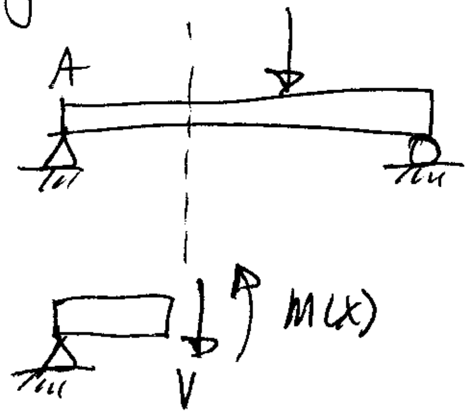
$y_A = 0$, $y_B = 0$



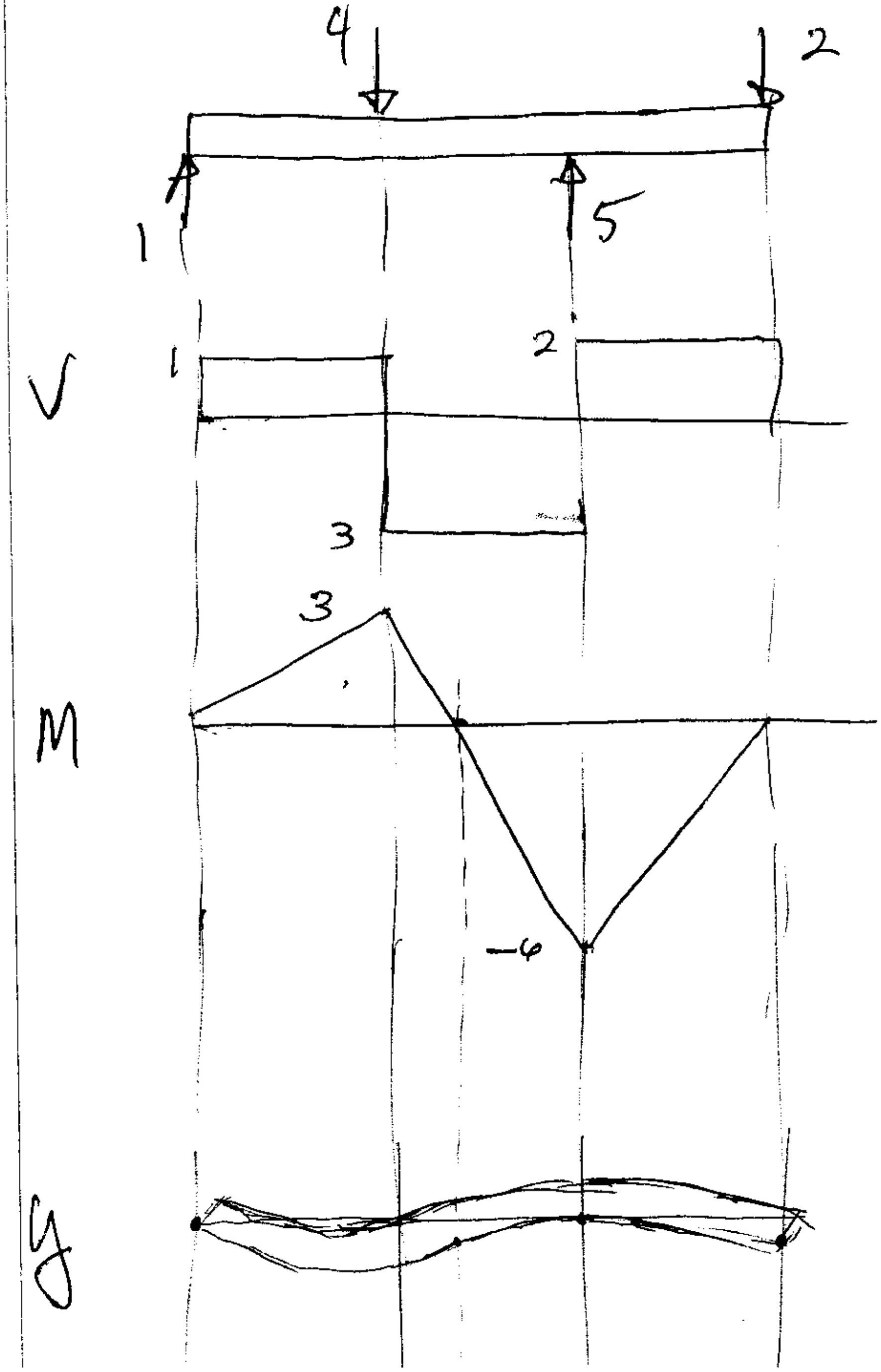
$y_A = 0$, $y_B = 0$

Determine M(x) by slicing through section where you want to know y(x) and

$\Sigma M_A = 0$



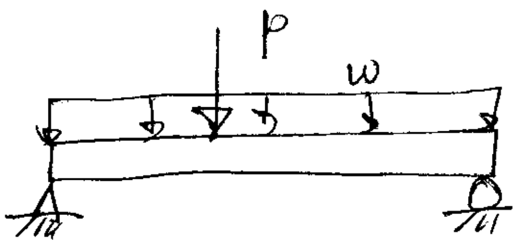
$m(x) = \_\_\_$



Superposition:

- Complicated loadings can be decomposed into several simple loads
- the deflection (y) and slope (θ) for the original problem can be obtained by adding the y's, θ's from the simple loading cases.
- This is facilitated by the use of tables

EX:



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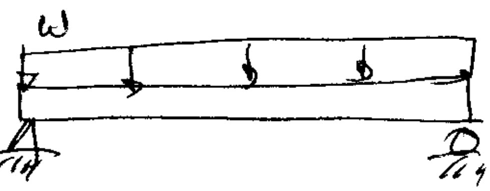
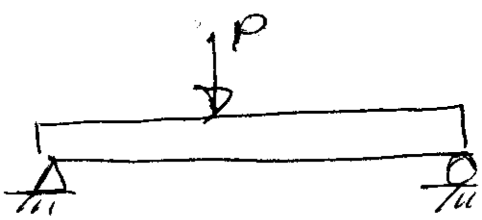


(I)

\+



(II)

$$y(x) = [y(x)]_{I} + [y(x)]_{II}$$

$$\theta(x) = [\theta(x)]_{I} + [\theta(x)]_{II}$$

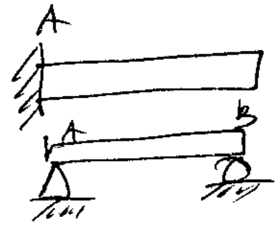
look these up in tables.

Statically Indeterminant Problems

Statics: $\underbrace{\sum F_y = 0 \,, \sum M = 0}_{\text{2 eqns}}$, $\cancel{\sum F_x = 0}$

2 eqns

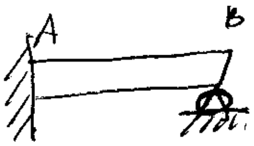
↳ ok for



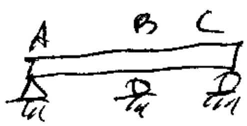
M_A, R_A

R_A, R_B

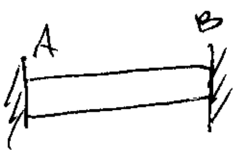
If we have "extra" supports, then beam is statically indeterminant:



3 reactions: M_A, R_A, R_B



: R_A, R_B, R_C



4 reactions: M_A, M_B, R_A, R_B

Use deflections, slopes to solve for one or more of reactions, then use statics to find the rest.

Procedure:

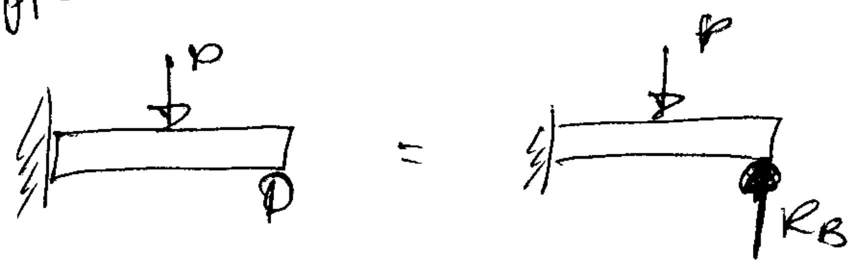
1) "Release" reactions one @ a time until beam is statically determinant.

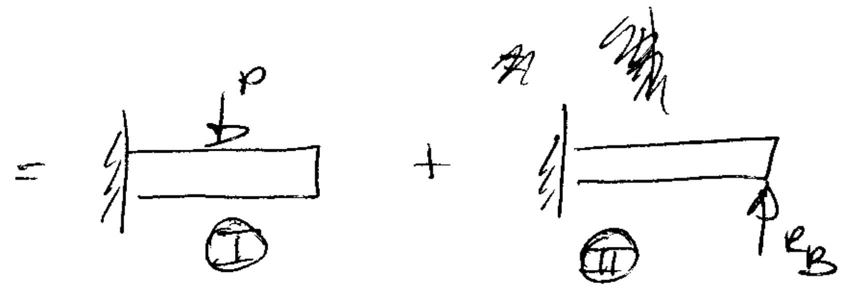
2) Replace each released reaction w/ a corresponding applied force/moment

3) use super position to find y and/or θ @ released support

4) set y and/or θ @ released support $= 0$, solve for unknown reaction(s)

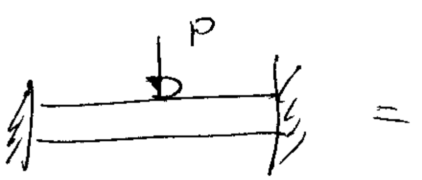
Ex:

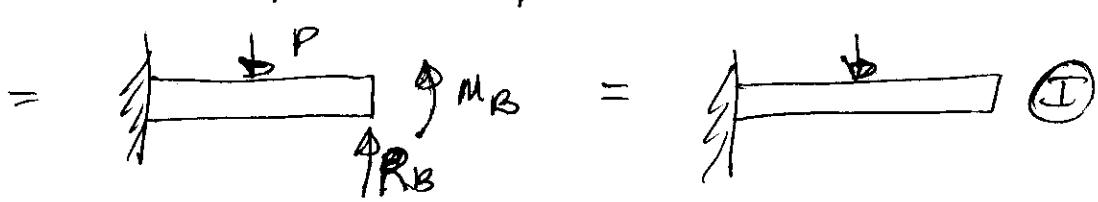


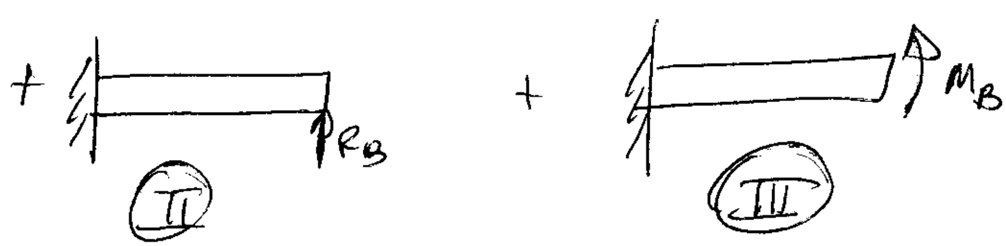


$$y_B = (y_B)_I + (y_B)_{II} = 0 \rightarrow \text{Solve for } R_B$$

EX:







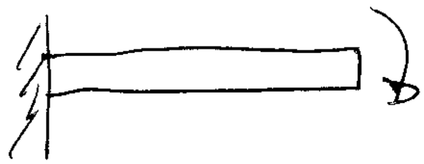
$$\left.\begin{aligned} y_B &= (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0 \\ \theta_B &= (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0 \end{aligned}\right\} \text{ solve for } R_B,\ \theta_B$$

Tips for using tables:

- Sometimes it is convenient to redraw beam looking @ "back side"

Ex:

backside:

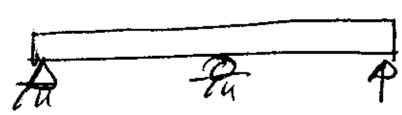


case 3

- decompose your beam into pieces that look like the table:

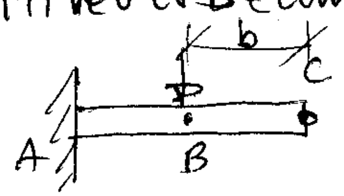
case 4

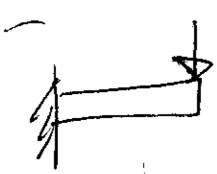
not



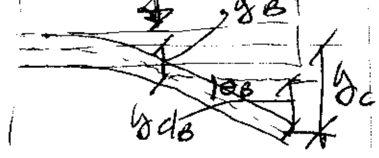
not in table

- Cantilever beams loaded in middle:





$\rightarrow y_B, \theta_B$



$y_c = y_B + y_{c/B}$

$y_{c/B} \rightarrow (\theta_B)b$

- Case 5

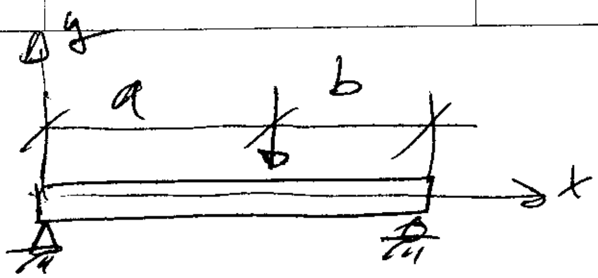


table gives $y(x) =$ ______ for $x \leq a$

for $y(x)$ for $x > a$:
replace x with $(L-x)$ and swap a, b
in eqns for $y(x)$ for $x<a$

- $x<a$

$$y(x) = \frac{Pb}{6EIL}\left[x^3 - (L^2-b^2)x\right]$$

$x>a$

$$y(x) = \frac{Pa}{6EIL}\left[(L-x)^3 - [L^2-a^2](L-x)\right]$$

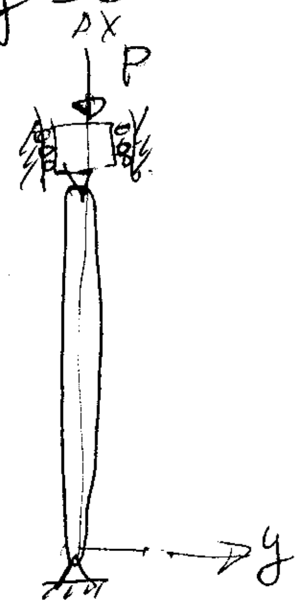
10 Buckling

Centric Axial loading of perfectly straight pin-ended column

Elastic Curve: $y(x) = A\sin\lambda x + B\cos\lambda x$

$$\lambda^2 = \frac{P}{EI}$$

→ Solution to Diff Eq: $\frac{d^2y}{dx^2} + \lambda^2 y = 0$



Apply B.C.s to find A, B

$x=0 \rightarrow y=0 \rightarrow B=0$

$x=L \rightarrow y = A\sin\lambda L = 0$

two options:

1) $A=0 \rightarrow y(x)=0$ uniform shortening

2) $\sin\lambda L = 0 \rightarrow \lambda L = \pi n = \sqrt{\frac{P}{EI}}$

$$P = \frac{\pi^2 n^2 EI}{L} \qquad n=1 \text{ (smallest } P)$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L}}$$

$P \geq P_{cr} \rightarrow$ column buckles

Can be extended to other end conditions:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$ → See overhead of end conditions.

Note:

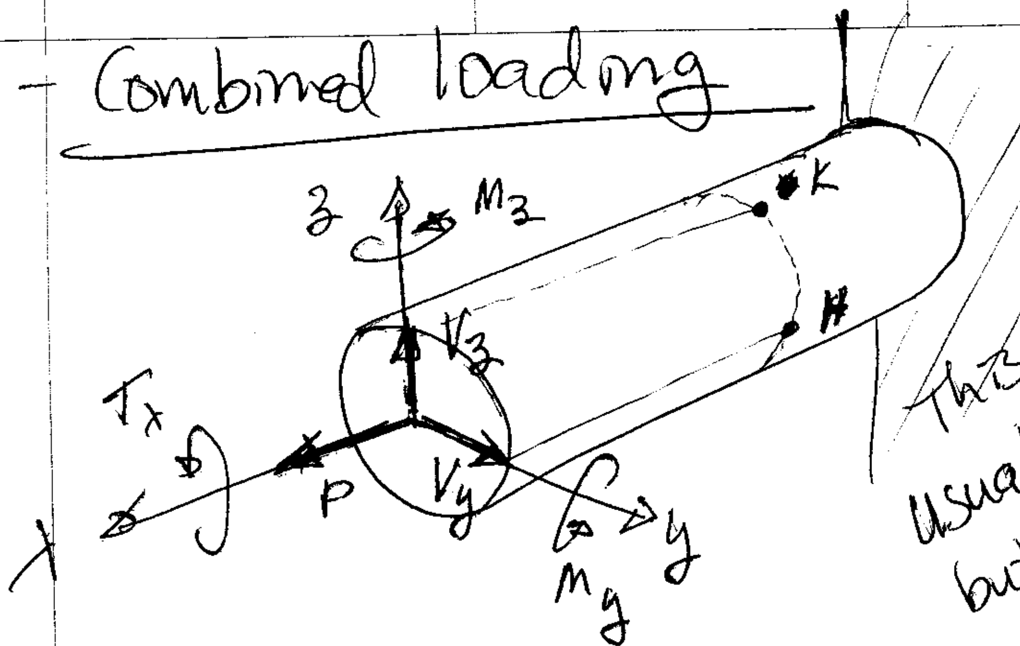
With Bending - we can choose which way we load the beam - so we can choose bending about strong axis (larger I)

→ for Buckling - column will buckle about weak axis - so you need to use smaller I ~~III~~ to determine P_{cr}.!!

Ways to Increase P_{cr}

- Increase E
- Increase I
- Decrease L_e — decrease L; change supports (i.e., pinned to fixed)

8 - Combined loading



This class: usually V_z or V_y but not both

Normal Stresses

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Pressure Vessels

$$\sigma_1 = \frac{p r_i}{t} = \text{hoop stress} = \text{circumferential stress}$$

$$\sigma_2 = \frac{p r_i}{2t} = \text{longitudinal stress} = \sigma_x$$

$\rightarrow$ @ H, $\sigma_1 = \sigma_z$, @ K, $\sigma_1 = \sigma_y$

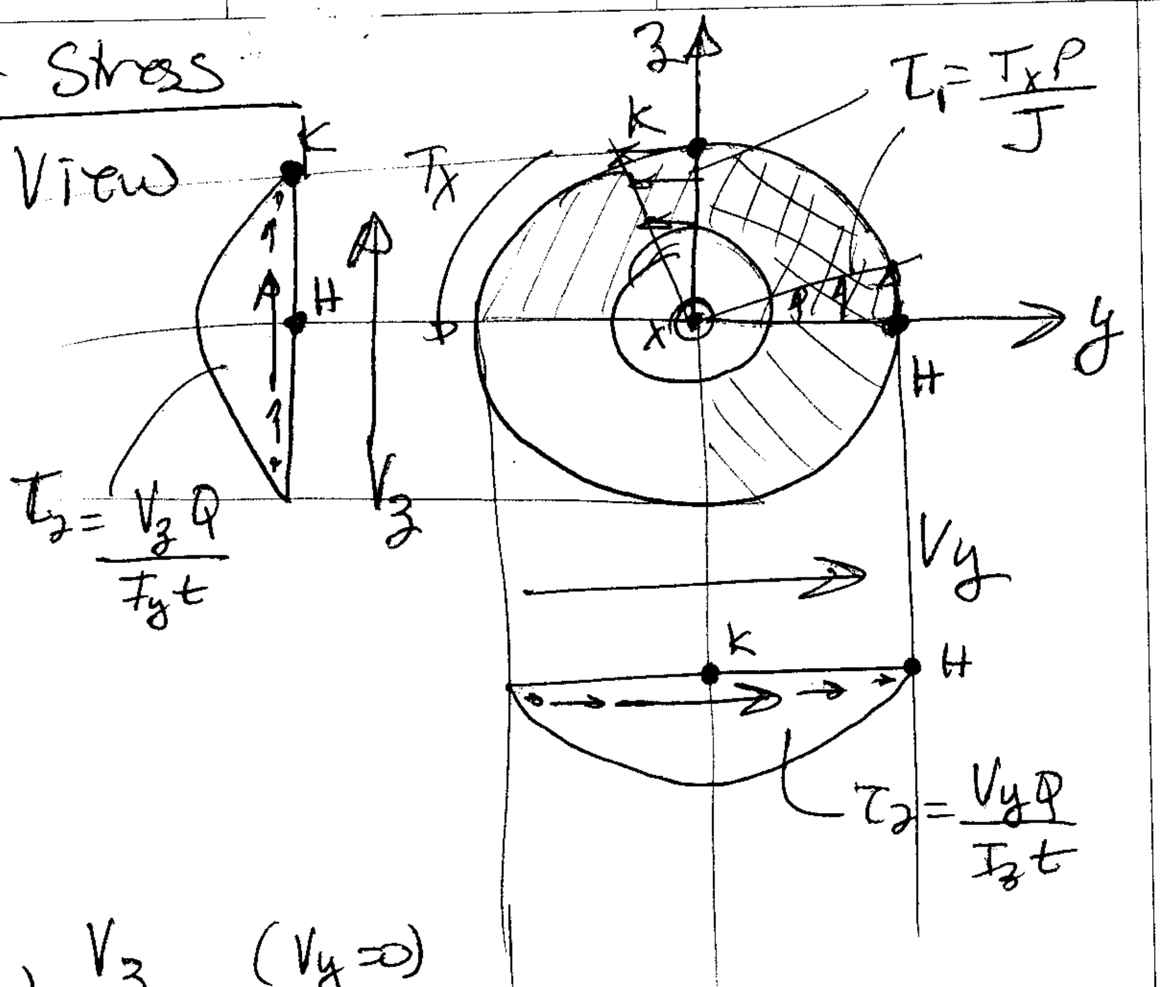
Shear Stresses

$$\tau = \tau_1 + \tau_2$$

$$\tau_1 = \frac{T_x \rho}{J}, \quad \tau_2 = \frac{V_z Q}{I_y t} \text{ or } \frac{V_y Q}{I_z t}$$

Shear Stress

End View



$t =$

a) T_x , V_z $(V_y = 0)$

K: $\tau_2 = 0$, $\tau_k = \tau_1 = \frac{T_x r_o}{J}$

H: $\tau_2 = \frac{V_z Q_{max}}{I_y t}$, $Q_{max} = \frac{2}{3}(r_o^2 - r_i^2)$

$Q = A\bar{y} = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2}{3} r^2$

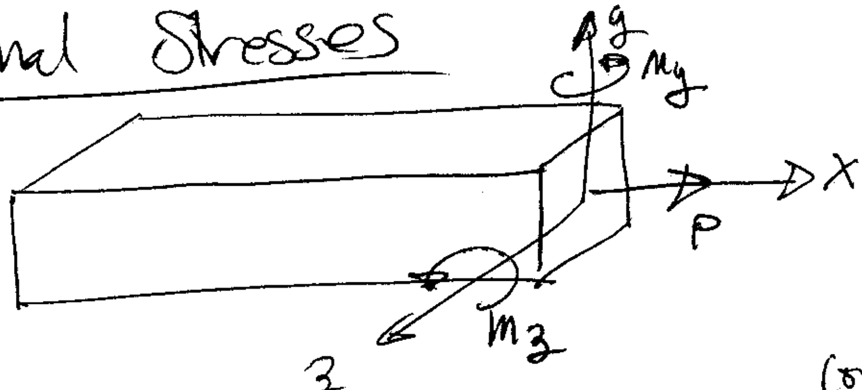
$\rightarrow \tau_H = \frac{T_x r_o}{J} + \frac{V_z Q_{max}}{I_y t}$

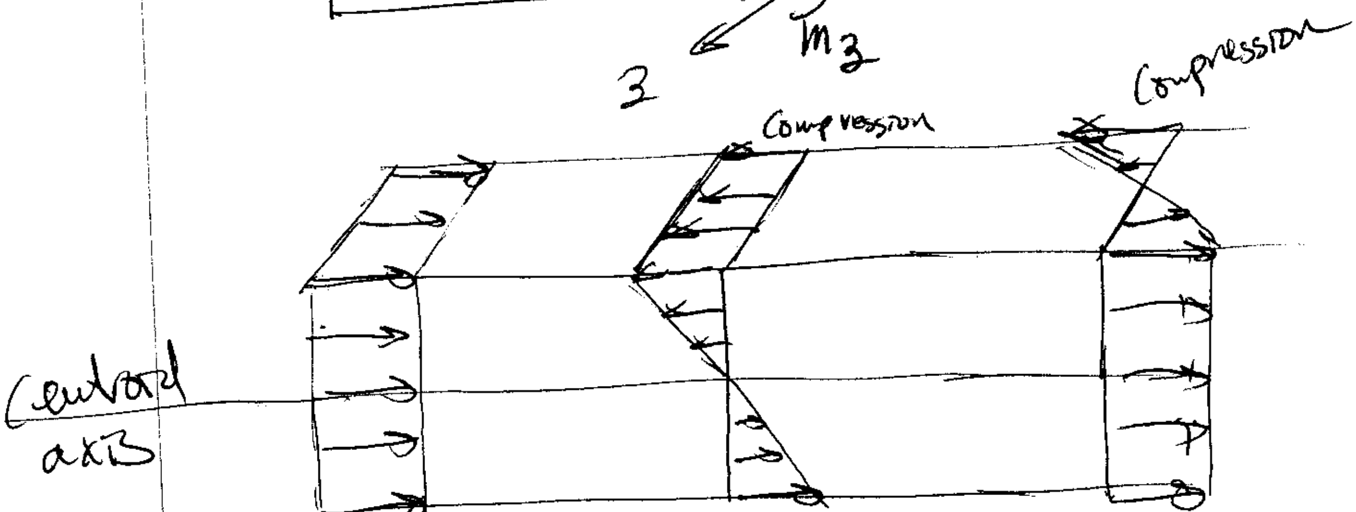
b) T_x , V_y ($V_z = 0$)

H: $\tau_2 = 0$, $\tau_H = \tau_1 = \frac{T_x r_o}{J}$

k: $\tau = -\frac{T_x r_o}{J} + \frac{V_y Q}{I_z t}$

Normal Stresses

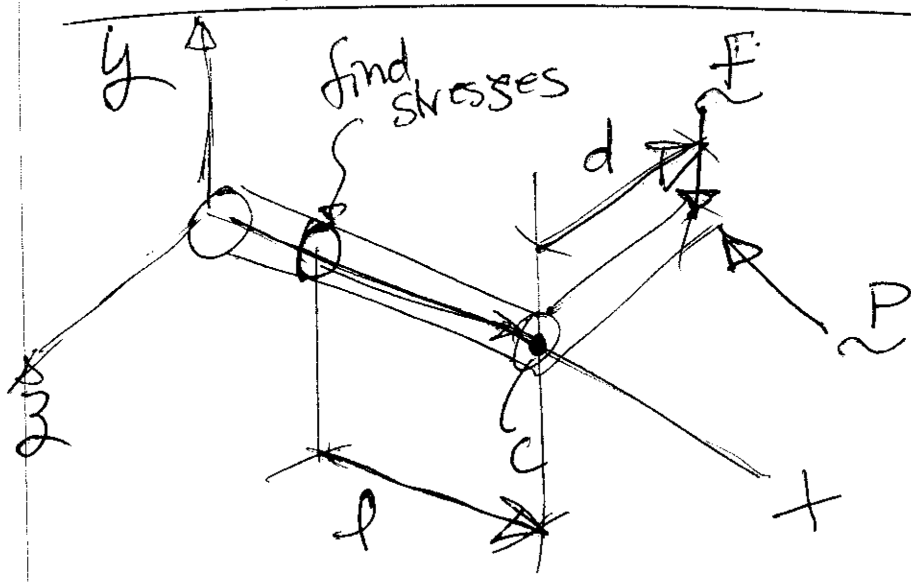




$\frac{P}{A}$ $-\frac{M_z y}{I_z}$ $+\frac{M_y z}{I_y}$

$y =$ } distance from centroid axis
$z =$ }

Tips for Combined loading



1) I.D. the location where you must find the stresses — this is your "beam" or "rod".

2) Move all the forces, moments to the centroid of the section of the beam
point C

2a) Put forces, lengths in vector format;

$\underset{\sim}{F} = -F\hat{j}$, $\underset{\sim}{P} = -P\hat{i}$

$\underset{\sim}{d} = -d\hat{k}$, $\underset{\sim}{l} = +l\hat{i}$

Axial load: $\underset{\sim}{P} = -P\hat{i}$

Shear: $\underset{\sim}{V_y} = -f\hat{j}$

Torsion: $\underset{\sim}{T_x} = \underset{\sim}{d} \times \underset{\sim}{f} = (-d\hat{k})\times(-f\hat{j})$
$= -df\hat{i}$

Bending: $\underset{\sim}{M_y} = \underset{\sim}{d} \times \underset{\sim}{P} = (-d\hat{k})\times(-P\hat{i}) = +dP\hat{j}$

$\underset{\sim}{M_z} = \underset{\sim}{l} \times \underset{\sim}{f} = (l\hat{i})\times(-f\hat{j}) = -lf\hat{k}$