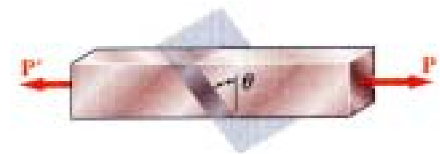
Clarkson University – ES222, Strength of Materials
Final Exam – Formula Sheet

Axial Loading

Normal Stress: $\sigma = \frac{P}{A}$ Splice joint: $\tau_{ave} = \frac{F}{A}$ Single shear: $\tau_{ave} = \frac{F}{A}$

Double shear: $\tau_{ave} = \frac{F}{2A}$ Bearing stress: $\sigma_b = \frac{P}{td}$

$$\sigma = \frac{P}{A_o}\cos^2\theta, \quad \tau = \frac{P}{A_o}\sin\theta\cos\theta$$



Factor of Safety = F.S. = $\frac{\text{ultimate load}}{\text{allowable load}}$

Stress and Strain – Axial Loading

Normal strain: $\varepsilon = \frac{\delta}{L}$ Normal stress: $\sigma = E\varepsilon$ Shear stress: $\tau = G\gamma$

Elongation: $\delta = \frac{PL}{AE}$ Rods in series: $\delta = \sum_i \frac{P_i L_i}{A_i E_i}$

Thermal elongation: $\delta_T = \alpha(\Delta T)L$ Thermal strain: $\varepsilon_T = \alpha(\Delta T)$

Poisson's ratio: $\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$

Generalized Hooke's Law:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

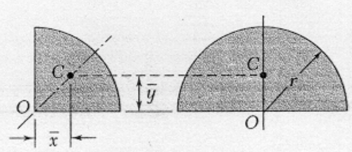
$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

Units: k = 10^3 M = 10^6 G = 10^9 Pa = N/m^2 psi = lb/in^2 ksi = 10^3 lb/in^2

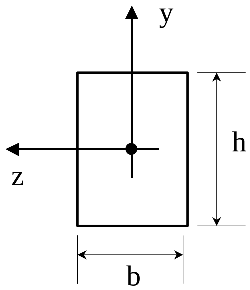
Coordinates of the Centroid: $\bar{x} = \frac{\sum_i x_i A_i}{\sum_i A_i}$ $\bar{y} = \frac{\sum_i y_i A_i}{\sum_i A_i}$

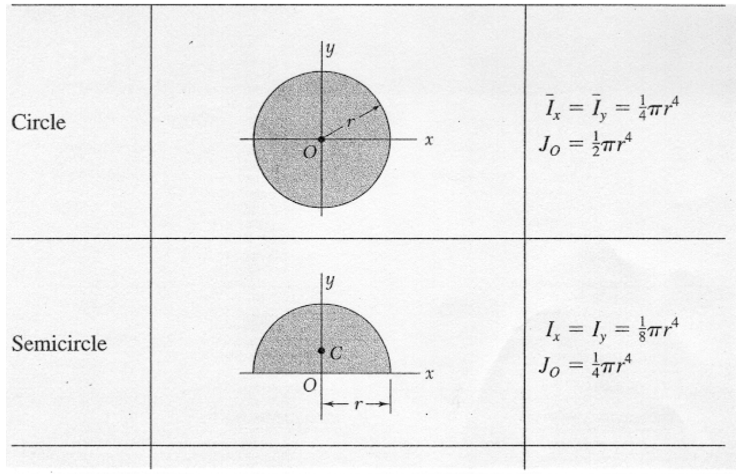
Parallel Axis Theorem: $I_{x'} = I_x + Ad^2$, where *d* is the distance from the *x*–axis to the *x*'–axis

Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area ←		$\bar{x} = 0$	$\bar{y} = \frac{4r}{3\pi}$	$A = \frac{\pi r^2}{2}$

$$I_z = \frac{1}{12} bh^3$$

$$I_y = \frac{1}{12} hb^3$$





<u>Torsion</u>:

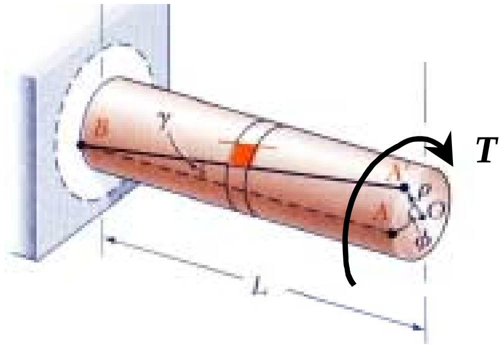
$$\gamma = \frac{\rho\phi}{L} \qquad \gamma_{\max} = \frac{c\phi}{L}$$

$$\tau = \frac{T\rho}{J} \qquad \tau_{\max} = \frac{Tc}{J} \qquad \tau = \gamma G$$

$$\phi = \frac{TL}{JG}$$

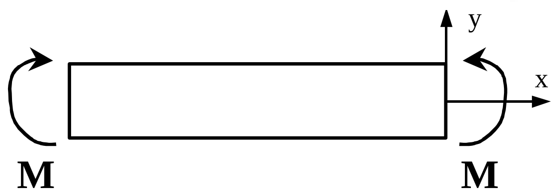
solid rod: $J = \frac{1}{2}\pi c^4$

hollow rod: $J = \frac{1}{2}\pi\left(c_o^4 - c_i^4\right)$



Rods in Series: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

<u>Pure Bending</u>:

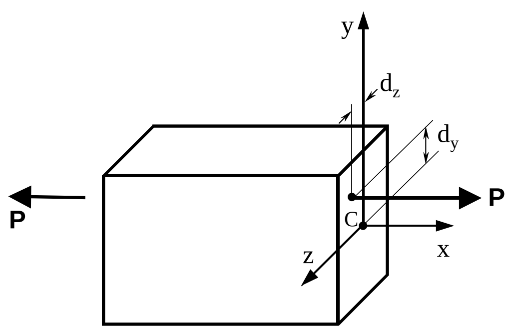


$$\sigma_x = -\frac{My}{I} \qquad \sigma_{\max} = \frac{Mc}{I} = \frac{M}{S}$$

$$\varepsilon_x = -\frac{y}{\rho} \qquad \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x \qquad \sigma = \varepsilon E \qquad \frac{1}{\rho} = \frac{M}{EI}$$

$\rho =$ radius of curvature

<u>General Eccentric Loading</u>:

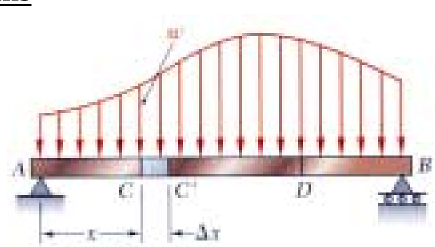


$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\vec{M}_z = \vec{d}_y \times \vec{P} \qquad \vec{M}_y = \vec{d}_z \times \vec{P}$$

<u>Shear and Bending Moment Diagrams</u>

(a) Internal forces
(positive shear and positive bending moment)



$$\frac{dV}{dx} = -w \quad \rightarrow \quad V_D - V_C = -\int_{x_c}^{x_d} w\,dx = -\text{(area under load curve between } C \text{ and } D\text{)}$$

$$\frac{dM}{dx} = V \quad \rightarrow \quad M_D - M_C = \int_{x_c}^{x_d} V\,dx = +\text{(area under shear curve between } C \text{ and } D\text{)}$$

<u>Shear Stress in Beams</u>

$\tau_{ave} = \frac{VQ}{It}$ $\qquad q = \frac{VQ}{I} =$ shear per unit length $\qquad Q = A\bar{y}$

<u>Stress Transformation</u>

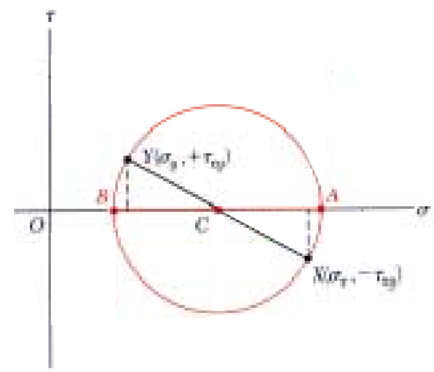
Principal stresses: $\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$

Principal planes: $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Planes of maximum in-plane shear stress: $\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$

Maximum in-plane shear stress: $\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} = R$

Corresponding normal stress: $\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$



<u>Thin Walled Pressure Vessels</u>

Cylindrical: Hoop stress = $\sigma_1 = \frac{pr}{t}$ $\qquad$ Longitudinal stress = $\sigma_2 = \frac{pr}{2t}$

Maximum shear stress (out of plane) = $\tau_{\max} = \sigma_2 = \frac{pr}{2t}$

Spherical: Principal stresses = $\sigma_1 = \sigma_2 = \frac{pr}{2t}$

Maximum shear stress (out of plane) = $\tau_{\max} = \frac{\sigma_2}{2} = \frac{pr}{4t}$

Deflections of Beams

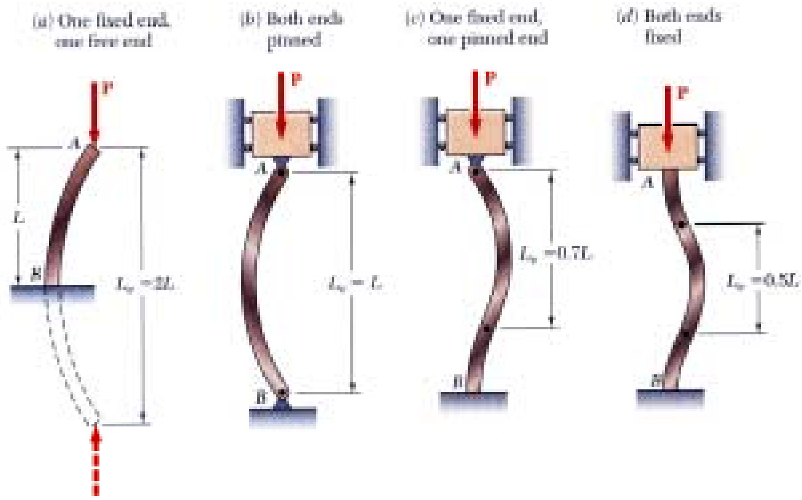
$$\frac{1}{\rho}=\frac{M(x)}{EI}=\frac{d^2y}{dx^2}$$

slope = $\theta(x)=\dfrac{dy}{dx}=\displaystyle\int\frac{M(x)}{EI}dx+C_1$

deflection = $y(x)=\displaystyle\int\theta(x)\,dx+C_2$ = elastic curve

Columns

$$P_{cr}=\frac{\pi^2EI}{L_e^2}$$



762 Appendix. D. Beam Deflections and Slopes

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y=\dfrac{P}{6EI}(x^3-3Lx^2)$
2		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y=-\dfrac{w}{24EI}(x^4-4Lx^3+6L^2x^2)$
3		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y=-\dfrac{M}{2EI}x^2$
4		$-\dfrac{PL^3}{48EI}$	$\pm\dfrac{PL^2}{16EI}$	For $x \le \frac{1}{2}L$: $y=\dfrac{P}{48EI}(4x^3-3L^2x)$
5		For $a > b$: $-\dfrac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m=\sqrt{\dfrac{L^2-b^2}{3}}$	$\theta_A=-\dfrac{Pb(L^2-b^2)}{6EIL}$ $\theta_B=+\dfrac{Pa(L^2-a^2)}{6EIL}$	For $x < a$: $y=\dfrac{Pb}{6EIL}[x^3-(L^2-b^2)x]$ For $x=a$: $y=-\dfrac{Pa^2b^2}{3EIL}$
6		$-\dfrac{5wL^4}{384EI}$	$\pm\dfrac{wL^3}{24EI}$	$y=-\dfrac{w}{24EI}(x^4-2Lx^3+L^3x)$
7		$\dfrac{ML^2}{9\sqrt{3}EI}$	$\theta_A=+\dfrac{ML}{6EI}$ $\theta_B=-\dfrac{ML}{3EI}$	$y=-\dfrac{M}{6EIL}(x^3-L^2x)$

For $x > a$, replace x with $(L-x)$ and interchange a with b.