Clarkson University

ES222 – Strength of Materials

Fall 2005 – Final Exam

Tuesday, December 13, 2005

Name: Solutions

Student ID No.: __________

Instructions:

Please check now and make sure your exam is complete. There should be 12 pages (including this coversheet and 4 formula sheets), with 10 problems.

Read the problems carefully and plan your time so as to gain the maximum number of points. The exam has 4 problems and is worth 100 points.

Please write neatly and clearly. Show all your work – simply listing answers is not sufficient. You will be given partial credit for work that leads to a logical conclusion.
Cross out any work that you do not wish to be considered.

The exam is closed book, closed notes. There are important formulas provided on the last 4 pages of the exam. You may not use your own formula sheet.

Calculators are not allowed (nor are they required).

You may remove the staple and/or attach extra pages of work, but make sure that you staple all the exam pages and extra work pages together before turning in the exam. Missing pages cannot be accepted after the exam.

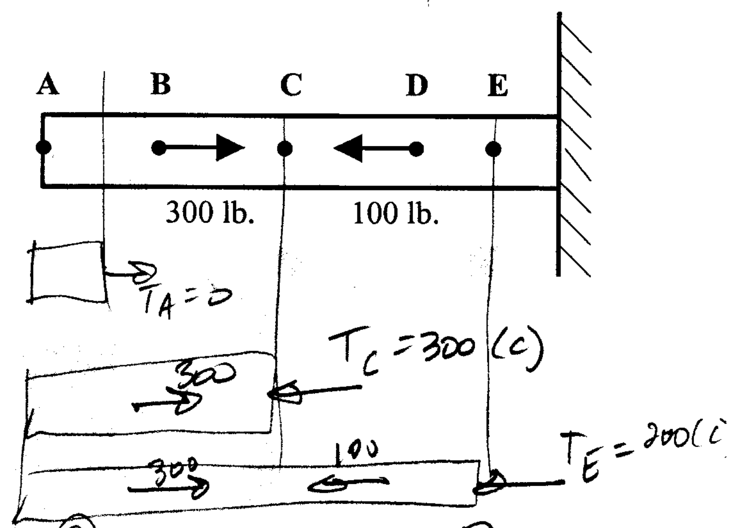
Grade:

1.	____ / 5	5.	____ / 6	9.	____ / 18
2.	____ / 6	6.	____ / 4	10.	____ / 18
3.	____ / 4	7.	____ / 20		
4.	____ / 4	8.	____ / 15		

Total: ____ / 100

Problem 1. (5 points)

Two external axial forces are applied at points B and D, as shown. For points A, C, and E, determine the internal axial force. Circle the correct magnitude, and circle either tension (T) or compression (C) if the internal force is not zero.



i) Point A: T or C ?
a) 0
b) 100 lb
c) 200 lb
d) 300 lb
e) 400 lb

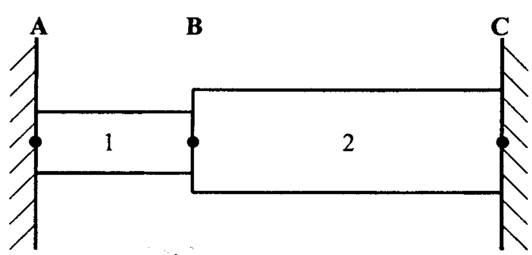
ii) Point C: T or C ?
a) 0
b) 100 lb
c) 200 lb
d) 300 lb
e) 400 lb

iii) Point E: T or C ?
a) 0
b) 100 lb
c) 200 lb
d) 300 lb
e) 400 lb

Problem 2. (0 points)

A rod consisting of two portions is shown below. The two portions are made of different materials. At room temperature, the rod just barely fits between the walls with no gap. The properties of the two rods are given below:

$A_1 = A$	$A_2 = 2A$
$L_1 = L$	$L_2 = 2L$
$E_1 = E$	$E_2 = 2E$
$\alpha_1 = \alpha$	$\alpha_2 = 2\alpha$

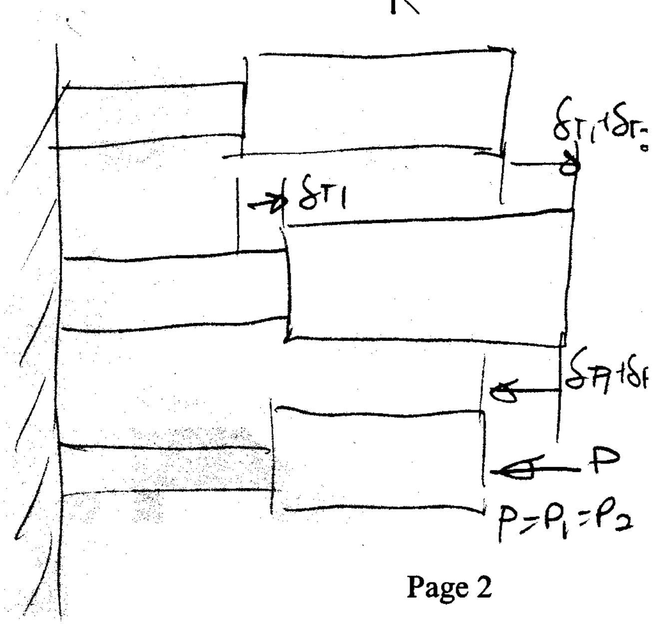


The temperature is increased, $\Delta T > 0$.

Circle all the true statements (there are several):

a) $\delta_1 = 0$
b) $\delta_{T1} > 0$
c) $\delta_{P1} > 0$
d) $\delta_2 = 0$
e) $\delta_{T1} < 0$
f) $\delta_{P1} < 0$
g) $\delta_1 + \delta_2 = 0$
h) $\delta_{T2} > 0$
i) $\delta_{P2} > 0$
j) $\delta_1 = \delta_2$
k) $\delta_{T2} < 0$
l) $\delta_{P2} < 0$

m) $P_1 = 0$
n) $P_1 > 0$
o) $P_2 = 0$
p) $P_1 < 0$
q) $P_1 + P_2 = 0$
r) $P_2 > 0$
s) $P_1 = P_2$
t) $P_2 < 0$

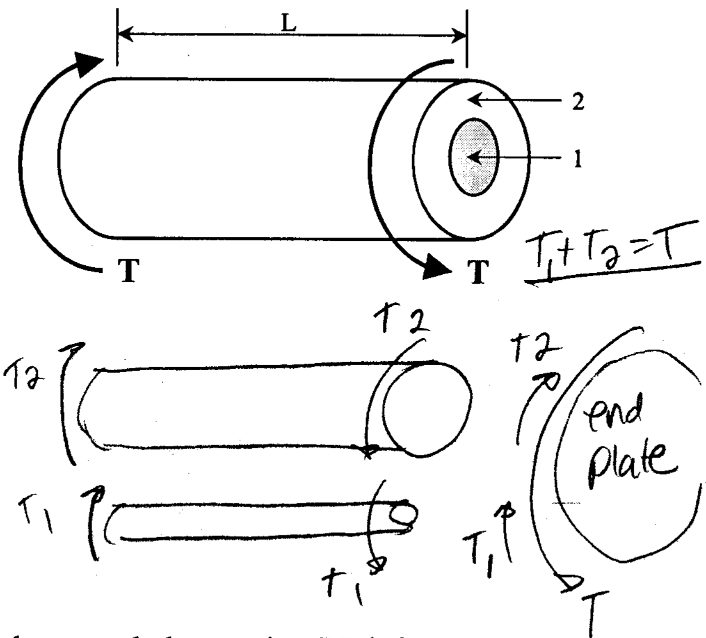


Problem 3. (4 points) $\rightarrow \phi = \phi_1 = \phi_2$

A shell of material 2 is fully bonded to a core of material 1, as shown below. A torque, T, is applied to the assembly, ~~such that~~ twist of the assembly is ϕ.

$J_1 = J \qquad J_2 = 2J$

$G_1 = G \qquad G_2 = 2G$



Circle the true statement:

a) $T_1 + T_2 = T$ and $\phi_1 + \phi_2 = \phi$

b) $T_1 + T_2 = T$ and $\phi_1 = \phi_2 = \phi$

c) $T_1 = T_2 = T$ and $\phi_1 + \phi_2 = \phi$

d) $T_1 = T_2 = T$ and $\phi_1 = \phi_2 = \phi$

Problem 4. (4 points)

The beam shown below is subjected to a four point bend test, such that portion BD is in pure bending. The cross section is shown below and point C is the centroid.

The formula for the magnitude of the maximum normal stress due to bending is: $\sigma = Mc/I$.

For the distances from C, shown below, the following is true: $\bar{y} < a < b < d < e < h$.

What is the proper value for c needed to determine the maximum **tensile** value of the normal stress?

a) a

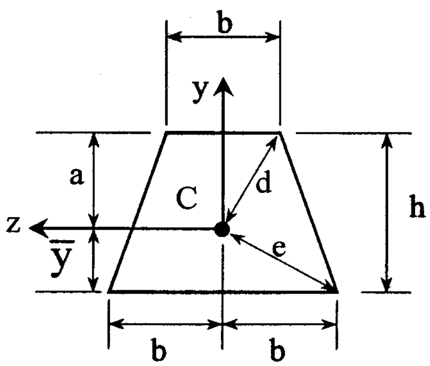
b) b

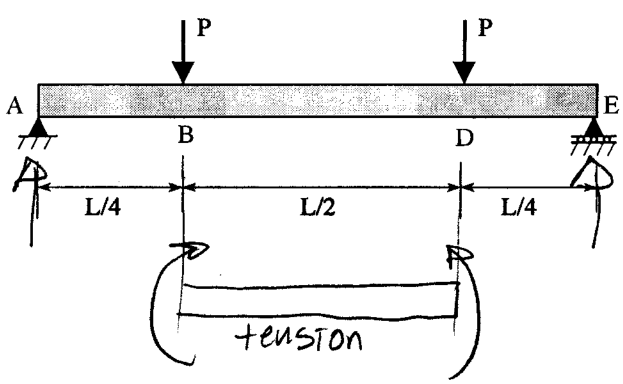
c) $\bar{y}$

d) d

e) e

f) h



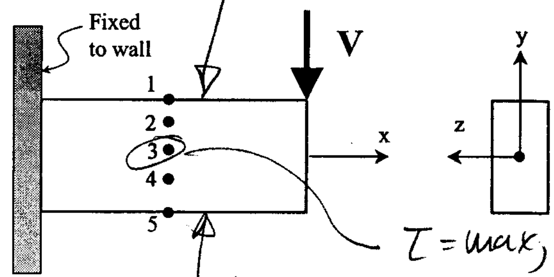


c = max distance in y-dir to bottom surface

$\tau = 0$, $\sigma_x = $ max, tension

Problem 5. (6 points)

A vertical shear force is applied to the end of the cantilever beam of rectangular cross section, shown to the right. Point 1 is on the top surface, point 3 at the centroid axis, point 5 is on the bottom surface, and points 2 and 3 are in between, as shown.



$\tau = $ max, $\sigma = 0$

$\tau = 0$, $\sigma_x = $ max, compr.

For point 1 and point 4, a small element of material, centered around each point, is shown below. Draw the normal stresses and shear stresses on each element of material. Make the length of each stress arrow approximately proportional to the magnitude of the stress.

1: $\sigma_x = $ max tension

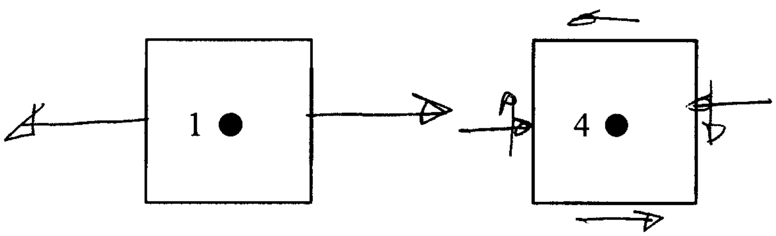
$\sigma_y = 0$

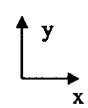
$\tau = 0$

4: $\sigma_x = $ compression (but not max)

$\sigma_y = 0$

$\tau \neq 0$, but not τ_{max}





Problem 6. (4 points)

Four pieces of wood are glued together to form a beam, show below. This beam is then subjected to a vertical shear force, which is slowly increased.

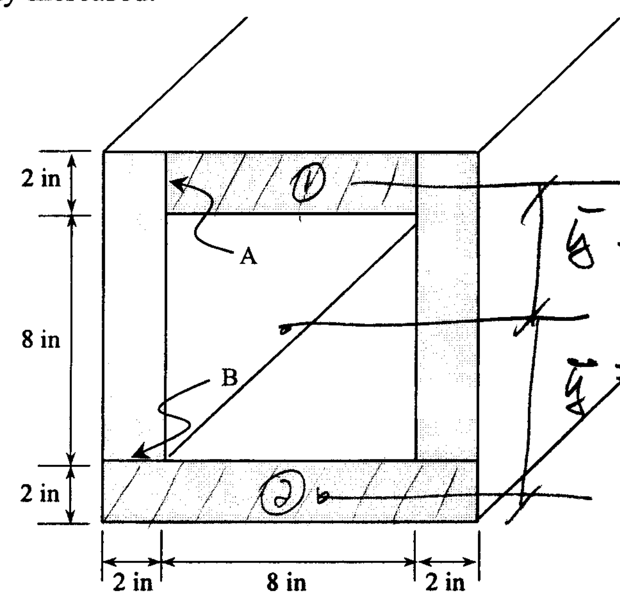
Circle the true statement:

a) Glue joint *A* will fail before glue joint *B*

b) Glue joint *B* will fail before glue joint *A*

c) Glue joints *A* and *B* will fail at the same time

d) There is insufficient information to decide



$\bar{y} = 5''$

$\bar{y} = 5''$

$\tau = \frac{VQ}{It}$, for A, B: I, t, V are all the same

$Q_A = Q_1 = A_1 \bar{y}_1 = (2)(8)(5)$

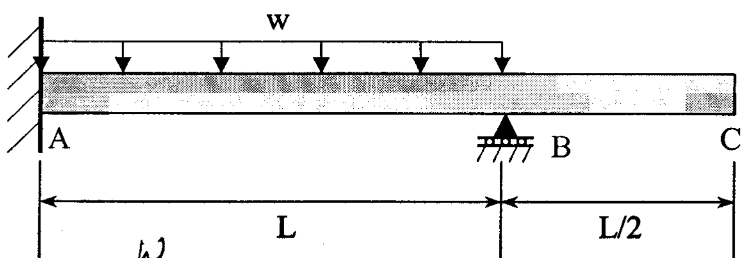
$Q_B = Q_2 = A_2 \bar{y}_2 = (2)(12)(5)$

$Q_B > Q_A \rightarrow \tau_B > \tau_A$

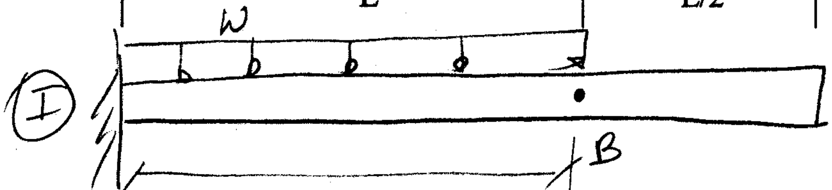
Problem 7. (20 points)

For the beam shown below, determine:

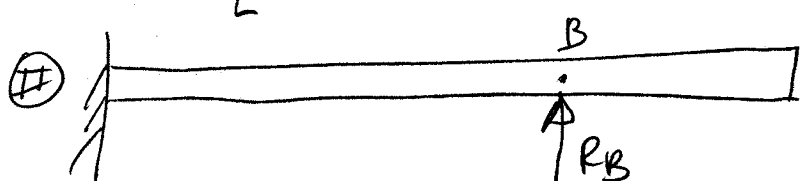
1) The reaction at B, R_B.
2) The deflection at point C, y_C.



$=$



$+$



A) Ignore tail (BC) + find $y_B \overset{0}{=}$, $\theta_B = \theta_C$

B) Add tail back + find y_C

I: $y_B = \dfrac{-wL^4}{8EI}$, $\theta_B = -\dfrac{wL^3}{6EI}$

II: $y_B = \dfrac{-(-R_B)L^3}{3EI}$, $\theta_B = -\dfrac{(-R_B)L^2}{2EI}$

$$y_B = \frac{-L^3}{24EI}\left(3wL - 8R_B\right) = 0$$

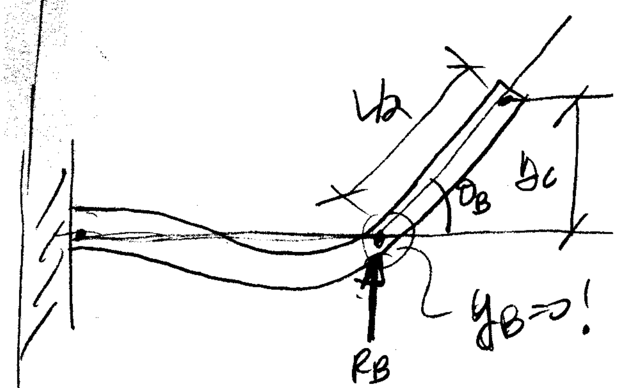
$$\boxed{R_B = \frac{3}{8}wL}$$

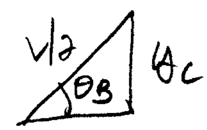
$$\theta_B = \frac{-L^2}{6EI}\left(wL - 3R_B\right) = \frac{-L^2}{6EI}\left[\frac{8wL}{8} - 3\left(\frac{3wL}{8}\right)\right]$$

$$\boxed{\theta_B = \frac{wL^3}{48EI}}$$

θ_B

Draw deflected shape;





$$\sin\theta_B = \frac{y_C}{L/2} \approx \theta_B$$

$$y_C = \frac{L}{2}\theta_B = \frac{L}{2}\left(\frac{wL^3}{48EI}\right)$$

$$\boxed{y_C = \frac{wL^4}{96EI}}$$

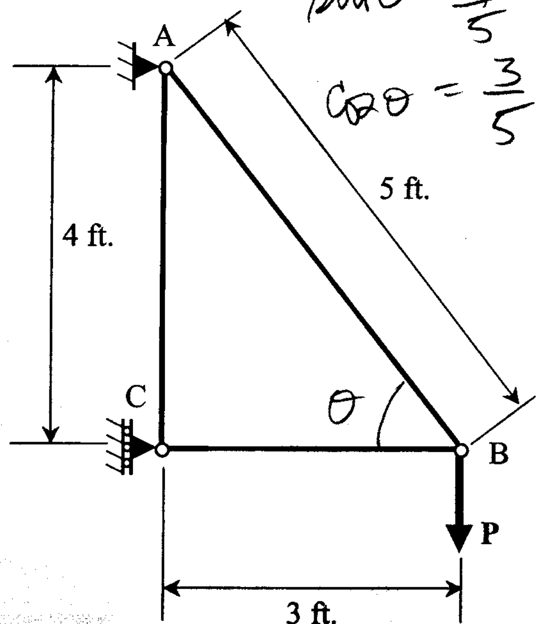
Problem 8. (15 points)

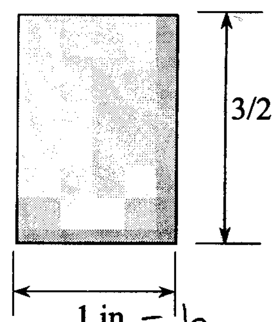
The truss shown below is loaded with a vertical force, P, at point B. The truss is to be composed of rods with rectangular cross section of 1 in. by 3/2 in., also shown below, and $E = 2 \times 10^6$ psi.

Considering only the condition for failure by buckling of rod BC, what is the maximum allowable load, P, which can be safely applied to the structure, if the factor of safety of 2 is required?

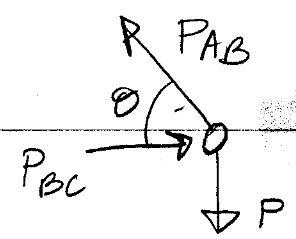
$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$





Joint B:



$$P_{cr} = \frac{\pi^2 EI}{L^2} = P_{BC}(FS)$$

$$P_{BC} = \frac{\pi^2 EI}{L^2(FS)}$$

choose smaller I:

$$I = \frac{1}{12} h b^3 = \frac{1}{4\,\cancel{12}}\left(\frac{\cancel{3}}{2}\right)(1)^3$$

$$I = \frac{1}{8}$$

$$L = 3, \; L^2 = 9$$

$$P_{BC} = \frac{\pi^2 E\left(\frac{1}{8}\right)}{9(2)} = \frac{\pi^2 E}{16(9)}$$

$$\Sigma F_y = +P_{AB}\sin\theta - P = 0$$

$$P_{AB} = \frac{P}{\sin\theta} = \frac{5}{4}P$$

$$\Sigma F_x = P_{BC} - P_{AB}\cos\theta = 0$$

$$P_{BC} = P_{AB}\cos\theta = \left(\frac{5}{4}\right)P\left(\frac{3}{5}\right) = \frac{3P}{4}$$

$$P_{BC} = \frac{3P}{\cancel{4}} = \frac{\pi^2 E}{4\,\cancel{16}(9)}$$

$$P = \frac{\pi^2(\cancel{2}\times 10^6)}{\cancel{2}\,\cancel{4}(27)} = \boxed{\frac{\pi^2 \times 10^6}{54} = P}$$

[redacted]

Problem 9. (18 points)

The cylindrical pressure vessel, shown below, is fixed to the floor. The internal pressure is p. A force, of magnitude F, is applied at point A, which is a distance a from the centroid axis of the vessel. Another force, of the same magnitude F, is applied at point B, which is a distance b from the centroid axis of the vessel. The dimensions of the pressure vessel are: r_o is the outer radius, r_i is the inner radius, and t is the thickness. We are interested in two points, H and K, which are located on the outer surface of the pressure vessel, at a distance of L from the end of the vessel.

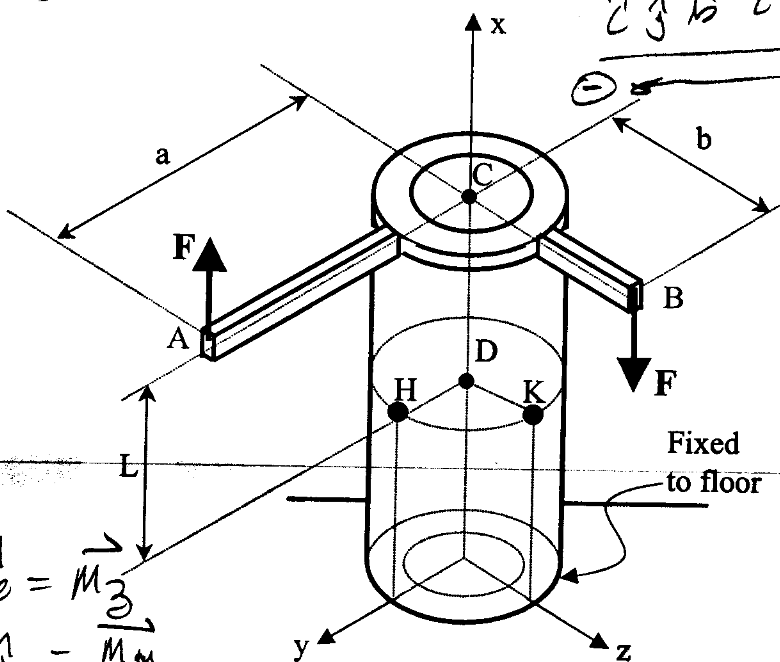
$y=0,\ z=+r_o$

a) Determine the stresses at point K in terms of the appropriate parameters: p, F, a, b, L, r_o, r_i, and t.

b) Determine the stresses at point H in terms of the appropriate parameters: p, F, a, b, L, r_o, r_i, and t.

$y=+r_o,\ z=0$

$\hat{i}\ \hat{j}\ \hat{k}\ \hat{i}\ \hat{j}$ (+) (−)



pressure:

$\sigma_z = \text{long stress} = \dfrac{p r_i}{2t} = \sigma_x$

$\sigma_1 = \text{hoop} = \dfrac{p r_i}{t}$

K: $\sigma_1 = \sigma_y$, H: $\sigma_1 = \sigma_z$

Forces + Moments

$P_x = +f - f = 0$

$\vec{a} \times \vec{F} = (+a\hat{j}) \times (+f\hat{i}) = -fa\hat{k} = \vec{M}_z$

$\vec{b} \times \vec{F} = (+b\hat{k}) \times (-f\hat{i}) = -fb\hat{j} = \vec{M}_y$

$\boxed{I_y = I_z = \frac{\pi}{4}(r_o^4 - r_i^4)}$

K: $\sigma_x = \cancel{\dfrac{P}{A}}^{\,0} - \dfrac{M_z \cancel{y}^{\,0}}{I_z} + \dfrac{M_y z}{I_y} + \dfrac{p r_i}{2t}$

K: $\boxed{\sigma_x = \dfrac{+(-fb)(r_o)}{\frac{\pi}{4}(r_o^4 - r_i^4)} + \dfrac{p r_i}{2t}}$

$\boxed{\sigma_y = \dfrac{p r_i}{t},\ \sigma_z = 0 \quad \tau = 0}$

H: $\sigma_x = \cancel{\dfrac{P}{A}}^{\,0} - \dfrac{M_z y}{I_z} + \dfrac{M_y \cancel{z}^{\,0}}{I_y} + \dfrac{p r_i}{2t}$

$\sigma_x = \dfrac{-(-fa)(r_o)}{\frac{\pi}{4}(r_o^4 - r_i^4)} + \dfrac{p r_i}{2t}$

$\sigma_y = 0,\ \sigma_z = \dfrac{p r_i}{t}$

$\tau = 0$

H

[redacted]

Problem 10. (18 points)

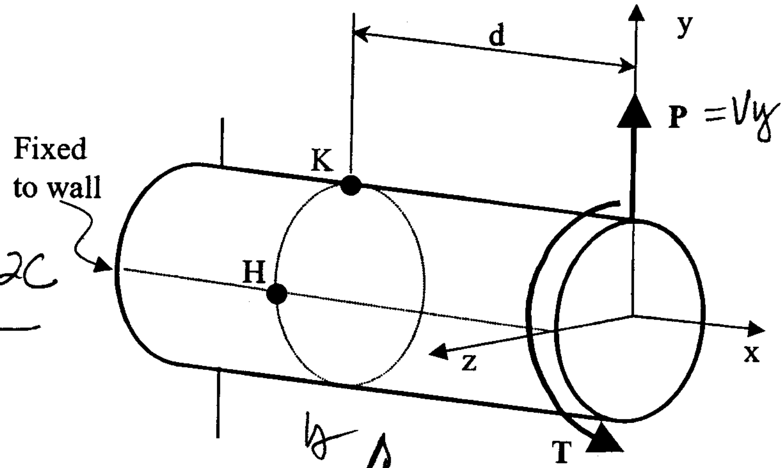
A solid cylindrical rod, of radius c, is fixed to the wall, as shown below. A torque, T, and force, P, are applied at the free end of the rod. We are interested in the stresses at points H and K, which are located on the outer surface of the rod, at a distance of d from the free end of the rod.

a) Determine the shear stress at point K in terms of the appropriate parameters, T, P, d, and c.
b) Determine the shear stress at point H in terms of the appropriate parameters, T, P, d, and c.

$$\tau = \pm\tau_1 \pm \tau_2$$

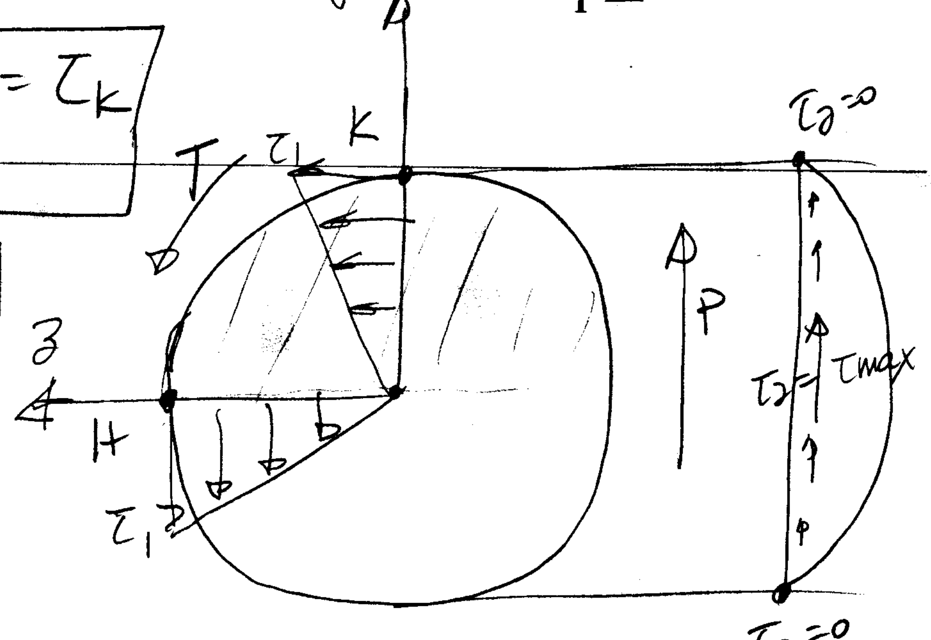
$$\tau_1 = \frac{Tc}{J}, \quad \tau_2 = \frac{PQ}{It}$$

$$J = \frac{\pi}{2}c^4, \quad I = \frac{\pi}{4}c^4, \quad t = 2c$$



K: $\tau_2 = 0$

$$\tau = \tau_1 = \frac{Tc}{\frac{\pi}{2}c^4} = \boxed{\frac{2T}{\pi c^3} = \tau_K}$$



H: $\tau = -\tau_1 + \tau_2$

$$Q = q_{max} = A\bar{y}$$

$$= \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3} = \frac{2c^3}{3}$$

$$\tau = \frac{-2T}{\pi c^3} + \frac{P(2c^3/3)}{\frac{\pi}{4}c^4(2c)}$$

$$= \boxed{\frac{-2T}{\pi c^3} + \frac{4P}{3\pi c^2} = \tau_H}$$