Clarkson University

ES222 – Strength of Materials

Spring 2007 – Exam I

Wednesday, February 7, 2007

Name: Solutions

Student ID No.: ______________________

Instructions:

Please check now and make sure your exam is complete. There should be 7 pages (including this coversheet), with 5 problems.

Read the problems carefully and plan your time so as to gain the maximum number of points. The exam has 5 problems and is worth 100 points.

Please write neatly and clearly. Show all your work – simply listing answers is not sufficient. Include free body diagrams, when appropriate. You will be given partial credit for work that leads to a logical conclusion. Cross out any work that you do not wish to be considered.

The exam is closed book, closed notes. There is a formula sheet on page 7 (including units information). You may NOT use your own formula sheet.

Calculators are not allowed (nor are they required).

You may remove the staple and/or attach extra pages of work, but make sure that you staple all the exam pages and extra work pages together before turning in the exam. Missing pages cannot be accepted after the exam.

Grade:

1. ________ / 10
2. ________ / 15
3. ________ / 25
4. ________ / 25
5. ________ / 25

Total: ________ / 100

Problem 1: (10 points)

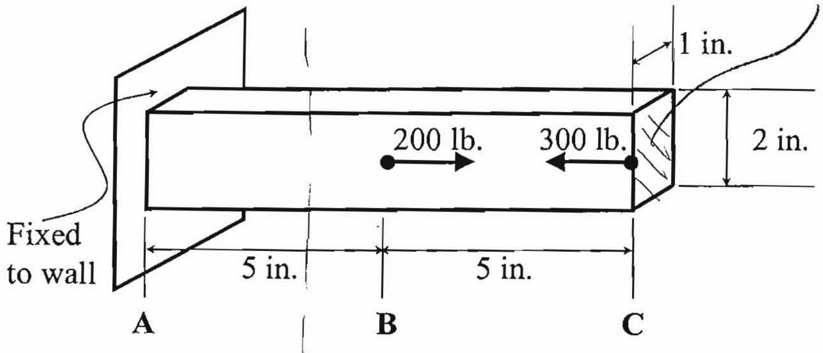
Two axial forces are applied to the bar shown below.

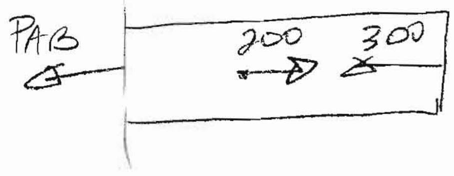
a) Determine the internal force, P_{AB} in portion *AB* of the bar. **Specify tension or compression.**

b) Determine the normal stress, σ_{AB}, in portion *AB* of the bar. **Specify tension or compression.**

Note: you must show all your work. Simply listing an answer is not sufficient.

$A = (1)(2) = 2\,in^2$





$$\Sigma F_x = -P_{AB} + 200 - 300 = 0$$

$$P_{AB} = -100$$

$$\boxed{P_{AB} = 100\,lb\ (C)}$$

$$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{100}{2} = 50\ psi$$

$$\boxed{\sigma_{AB} = 50\ psi\ (C)}$$

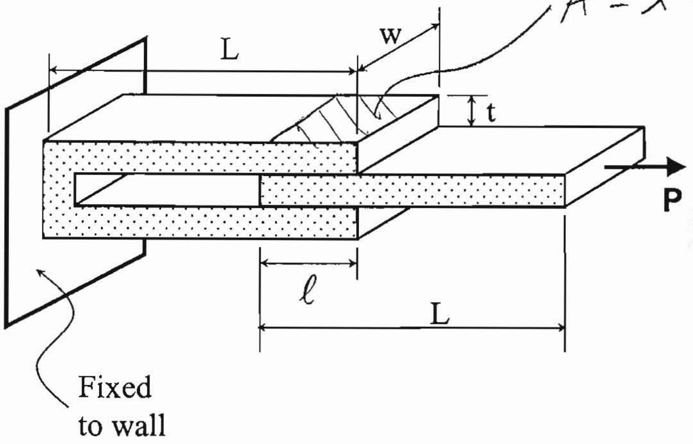
Problem 2: (15 points)

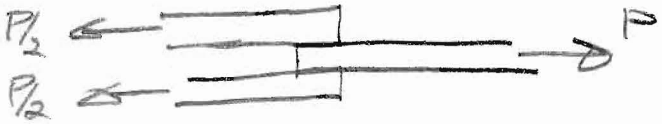
The U-shaped part is attached to the wall at the left. An overlapping glue joint, of length $\ell = 3$ in. is used to joint the U-shaped part to a flat bar. A tensile axial load of P is applied to the flat bar. The thicknesses of the U-shaped part and the flat bar are both t and the widths of both are w. The Lengths of the U-shaped part and the flat bar are both L.

$t = 1$ in. $\quad w = 4$ in. $\quad \ell = 3$ in. $\quad L = 12$ in.

For the overlapping glue joint, the ultimate shear stress for the glue is 200 psi. If a factor of safety is 2.0 required, what is the maximum allowable load, P, which can be applied?

$A = \ell w = (3)(4) = 12 \text{ in}^2$





$$\tau_{all} = \frac{\tau_y}{fs} = \frac{200}{2.0} = 100 \text{ psi}$$

$$\tau = \frac{P}{2A} = \frac{P}{2(12)} = \tau_{all} = 100$$

$$\boxed{P = 2400 \text{ lb.}}$$

Problem 3: (25 points)

A composite axial rod is fixed to the two walls. The entire rod is composed of the same material, with Young's modulus = *E*. The cross sectional areas of portions *AC* and *CD* are given below. The lengths of the portions are shown below. Forces are applied at points *B* and *C*, as shown below: $P_B = 200$ lb., $P_C = 100$ lb.

$E_{AC} = E_{CD} = E = E_1 = E_2 = E_3$

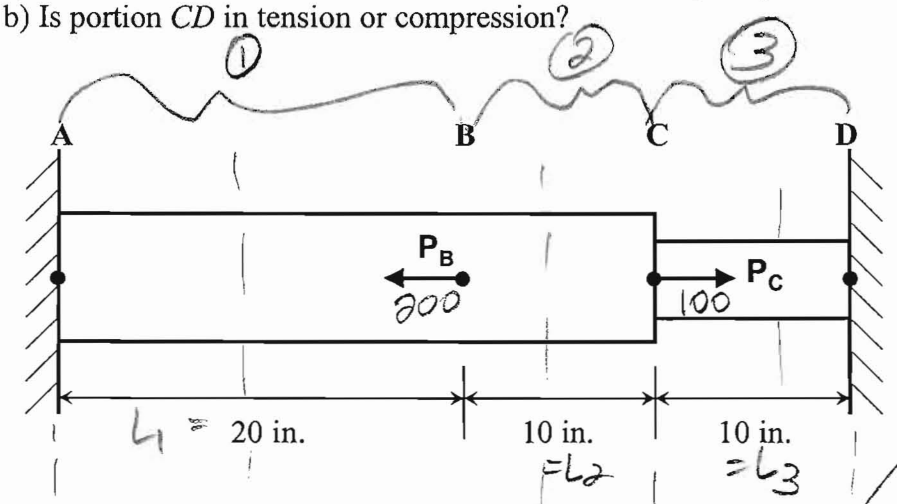
$\delta_1 + \delta_2 + \delta_3 = 0$

$A_1 = A_2 = A_{AC} = 2$ in.^{2} $\quad$ $A_{CD} = 1$ in.$^2 = A_3$

$P_B = 200$ lb. $\quad$ $P_C = 100$ lb.

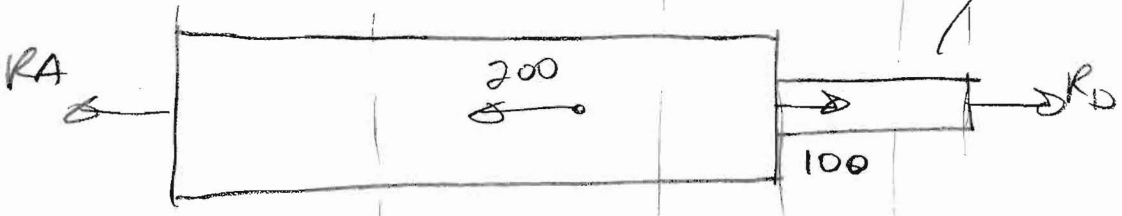
a) Determine the reactions at the walls, R_A and R_D. **Specify the direction for each (← or →).**

b) Is portion *CD* in tension or compression?

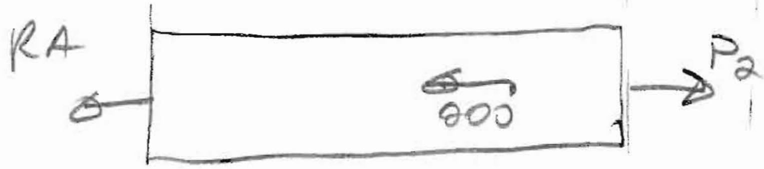


$\Sigma f_x = -R_A - 200 + 100 + R_D = 0$

$\boxed{R_D = R_A + 100}$

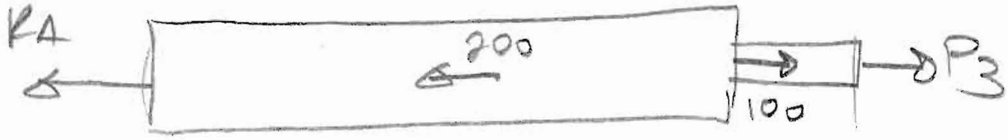


$\Sigma f_x = -R_A + P_1 = 0$, $\boxed{P_1 = R_A}$



$\Sigma f_x = -R_A - 200 + P_2 = 0$

$\boxed{P_2 = R_A + 200}$



$\Sigma f_x = -R_A - 200 + 100 + P_3 = 0$) $\boxed{P_3 = R_A + 100}$

$$\delta_1 + \delta_2 + \delta_3 = 0$$

$$\frac{P_1 L_1}{A_1 \cancel{E}} + \frac{P_2 L_2}{A_2 \cancel{E}} + \frac{P_3 L_3}{A_3 \cancel{E}} = 0$$

$$\frac{R_A\,(2\cancel{0})}{2} + \frac{(R_A + 200)(1\cancel{0})}{2} + \frac{(R_A + 100)(1\cancel{0})}{1} = 0$$

$$2R_A + R_A + 200 + 2R_A + 200 = 0$$

$$5R_A = -400$$

$$R_A = -\frac{400}{5} = -80, \quad \boxed{R_A = 80\text{ lb} \rightarrow}$$

$$R_D = R_A + 100 = -80 + 100, \quad \boxed{R_D = 20\text{ lb} \rightarrow}$$

b) $P_3 = R_A + 100 = -80 + 100 = 20 > 0,$
we drew it in tension,
so P_3 is tension

$$\boxed{\text{CD is in tension}}$$

Problem 4: (25 points)

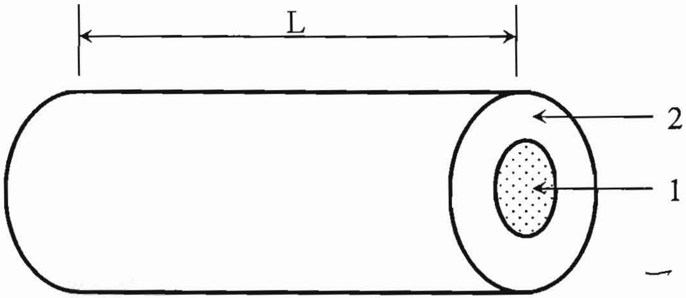
An outer shell of material 2 is fully bonded to a core of material 1. The material properties and cross sectional areas are given below.

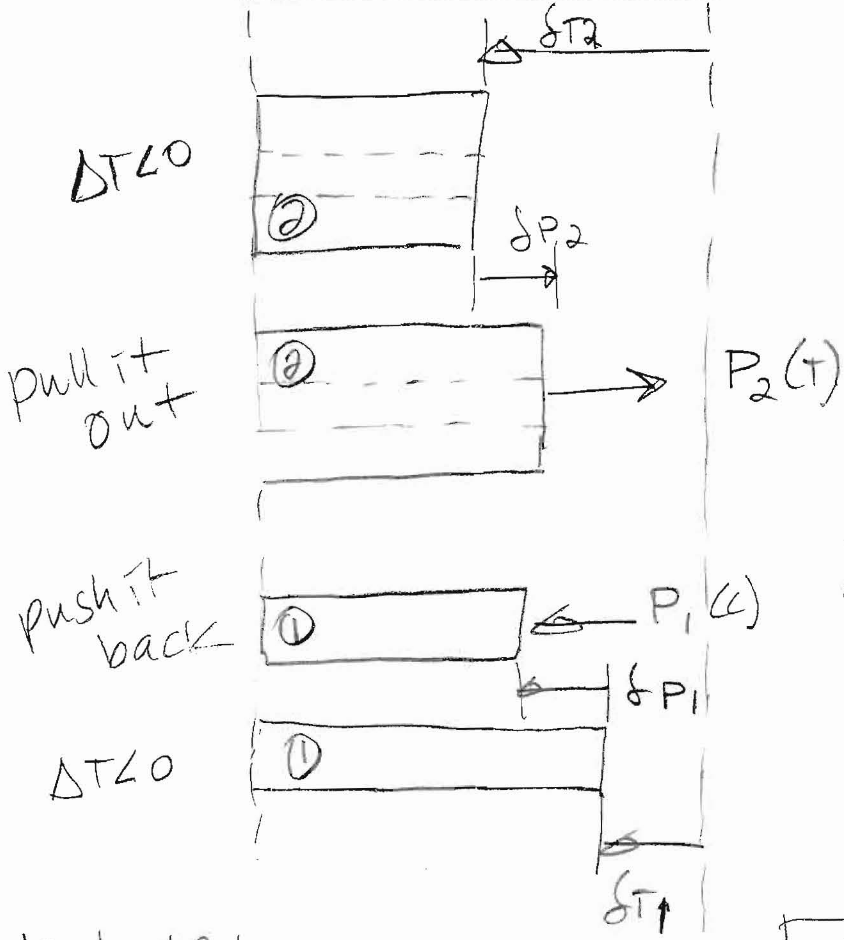
Core	Shell
$A_1 = A$	$A_2 = 2A$
$E_1 = E$	$E_2 = 2E$
$\alpha_1 = \alpha$	$\alpha_2 = 2\alpha$

→ $\alpha_2 > \alpha_1$ so 2 wants to get shorter than 1

The temperature is ***reduced*** by 100°. both get shorter

Determine the normal force, P_2, in the shell, in terms of A, E, L and α. (Note: your answer may not need to include all of the parameters A, E, L and α). **Specify tension or compression.**





$$\delta_1 = \delta_2$$

$$-\delta_{T2} + \delta_{P2} = -\delta_{T1} - \delta_{P2}$$

$$-\alpha_2(\Delta T)\cancel{L} + \frac{P\cancel{L}}{A_2E_2} = -\alpha_1(\Delta T)\cancel{L} - \frac{P\cancel{L}}{A_1E_1}$$

$$-(2\alpha)(100) + \frac{P}{(2A)(2E)} = -\alpha(100) - \frac{P}{AE}$$

$$\frac{P}{AE}\underbrace{\left(\frac{1}{4}+1\right)}_{\frac{5}{4}} = 100(2\alpha - \alpha)$$

$$P = 100\alpha AE\left(\frac{4}{5}\right)$$

$$P = 80\alpha AE$$

$$|P_2| = |P_1| = P$$

$$\boxed{P_2 = 80\alpha AE \text{ (T)}}$$

Problem 5: (25 points)

The piece of material, shown below, has undeformed dimensions of: $L_x = 3$ in., and $L_y = 2$ in., and $L_z = 1$ in. The Young's modulus is $E = 1 \times 10^6$ psi, and Poisson's ratio is $\nu = 1/4$. Two loads, P_x and P_y are applied, as shown below.

$L_x = 3$ in.	$E = 1 \times 10^6$ psi
$L_y = 2$ in.	$\nu = 1/4$
$L_z = 1$ in.	
$P_x = 300$ lb.	$P_y = 600$ lb.

$$A_1 = L_z L_y = (1)(2) = 2 \text{ in}^2$$

$$A_2 = L_z L_x = (1)(3) = 3 \text{ in}^2$$

a) Determine the change length in the x–direction, δ_x, due to the applied forces (specify longer, shorter, or zero).

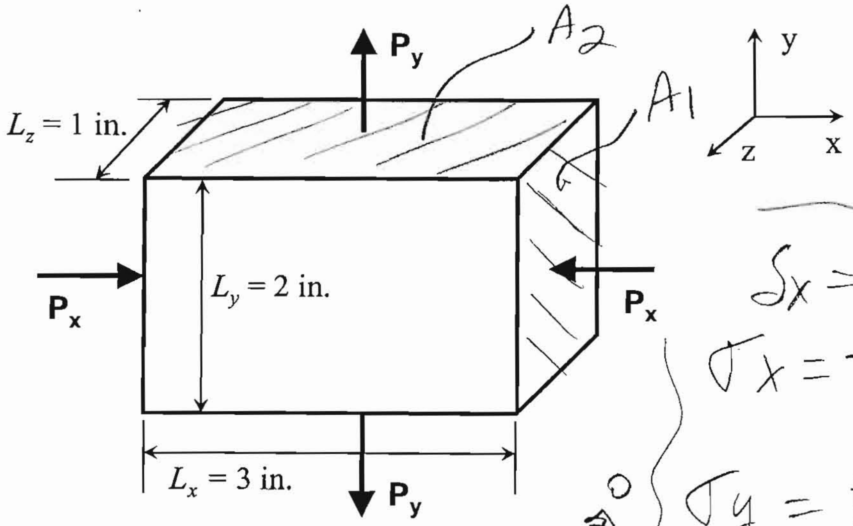
b) The change in length in the z–direction is defined as δ_z. Circle the correct answer:

1) Due to the applied forces, $\delta_z = 0$

[✓ 2) Due to the applied forces, $\delta_z \neq 0$]

since $\delta_z = \varepsilon_z L_z$

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \neq 0$$



$$\delta_x = \varepsilon_x L_x$$

$$\sigma_x = \frac{-P_x}{A_1} = -\frac{300}{2} = -150 \text{ psi}$$

$$\sigma_y = \frac{+P_y}{A_2} = \frac{+600}{3} = +200 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\left(\frac{\sigma_y}{E}\right) - \nu\left(\cancel{\frac{\sigma_z}{E}}\right) \quad 0$$

$$= \frac{-150}{10^6} - \left(\frac{1}{4}\right)\frac{(200)}{10^6} = -200 \times 10^{-6}$$

$$\delta_x = \varepsilon_x L_x = -200 \times 10^{-6}\,(3) = \boxed{-600 \times 10^{-6} \text{ in} = \delta_x}$$

$$\boxed{\text{gets shorter}}$$