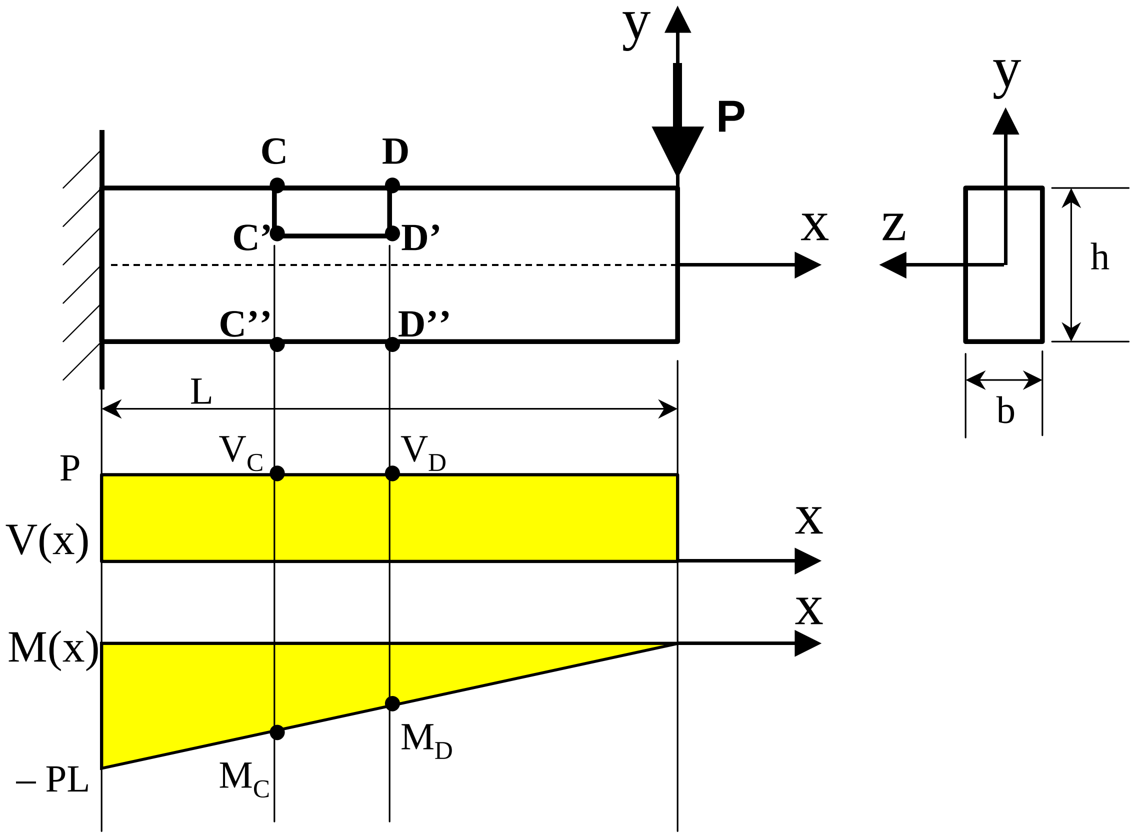
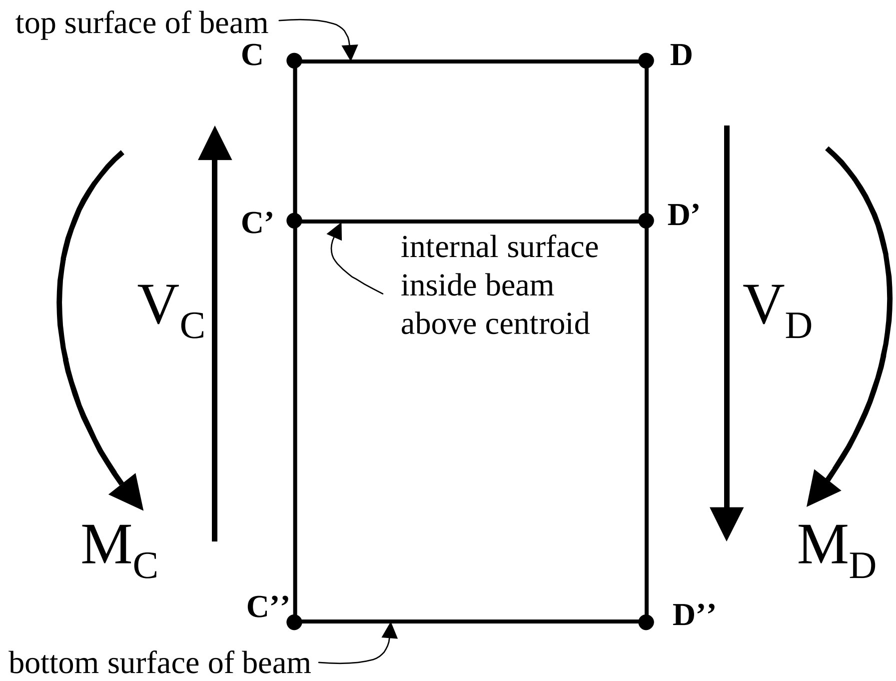
Shear Stresses in Beams

A shear force, P, is applied to the end of a cantilever beam.

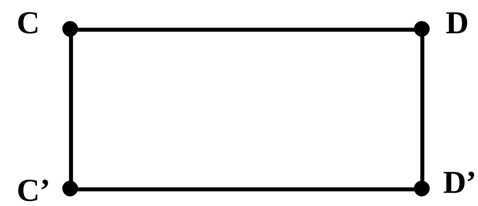


What is the relationship between the moment at C, M_C, and the moment at D, M_D?

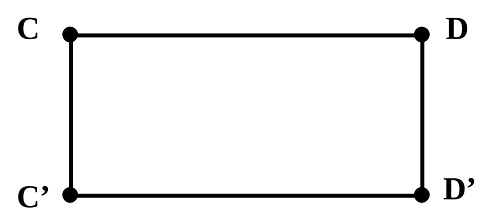
Shear force and bending moment at sections C and D:

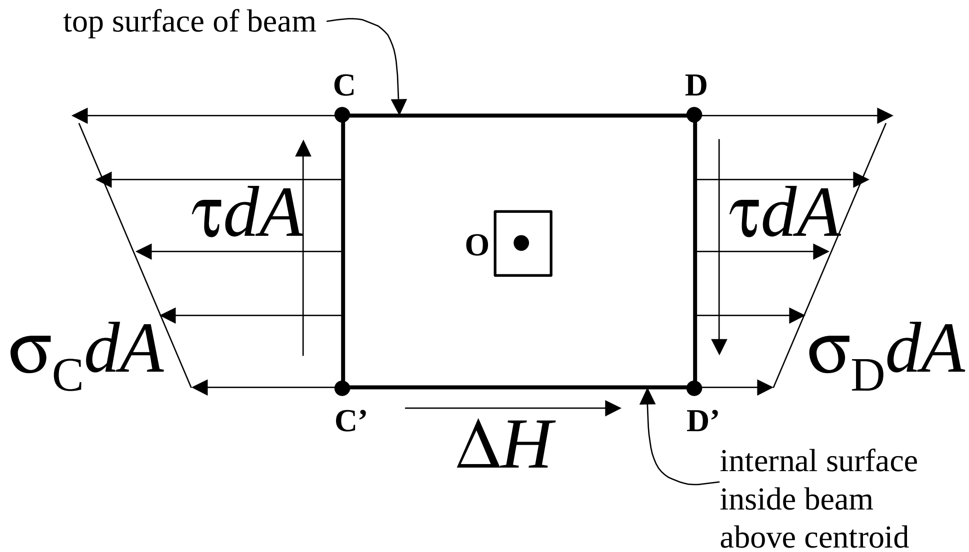


What are the normal stresses that act on the faces (CD, CC’, C’D’, DD’) of the material shown below?



What are the shear stresses that act on the faces (CD, CC’, C’D’, DD’) of the material shown below?





Recall that $\Sigma F_x = 0$:

$$\Sigma F_x = \Delta H + \int \sigma_D dA - \int \sigma_C dA = 0$$

$$\Delta H = \int \frac{(M_D - M_C)}{I} y dA = \frac{(M_D - M_C)}{I} \int y dA$$

Note: $Q = \int y dA$ = first moment of area

$$\Delta H = (M_D - M_C)\frac{Q}{I} = \Delta M \frac{Q}{I}$$

$$\frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{I}$$

$$q = \frac{dH}{dx} = \frac{dM}{dx}\frac{Q}{I} = V\frac{Q}{I}$$

$q = \dfrac{VQ}{I}$ = shear force per unit length (a.k.a. shear flow)

Shear force per unit length

$$q = \frac{VQ}{I}$$

(Used in nail and bolt problems)

Shear Stress

$$\tau = \frac{\Delta H}{\Delta A} = \frac{\Delta H}{t\Delta x} = \frac{q}{t} = \frac{VQ}{It}$$

$$\tau = \frac{VQ}{It}$$

where t = thickness of section where you want to find the shear stress (in our example, this is b)

(Used to find shear stress in material or in glue joints)

<u>Notes:</u>

I is for the entire cross section

- This is the same I you would use for the bending problem
- For rectangular cross sections, $I = \frac{1}{12}bh^3$

Q is for the piece that would fall off if shear is too big

- This depends on where you want to find the q or τ
- For rectangular cross sections, $Q = A\overline{y}$