

Clarkson University
Department of Mechanical and Aeronautical Engineering
AE/ME 455 Mechanical Vibrations
Spring 2005 – OPTIONAL QUIZ
Thursday, March 3, 2005

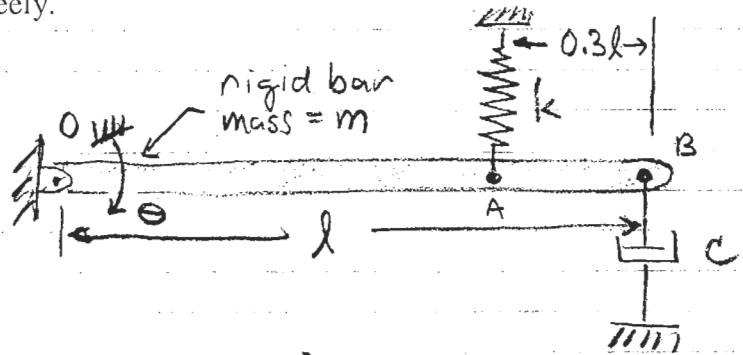
Name: SOLUTIONS

Student No.: _____

Instructions: Read the problems carefully and space your time so as to gain the maximum number of points of credit. You will be given partial credit only for those steps in a solution that are leading to a logical conclusion. Please write neatly and clearly and make clear unambiguous sketches; cross out any work you do not wish to be considered. This quiz has 2 main problems with multiple parts worth 40 points total. The quiz is CLOSED BOOK and CLOSED NOTES. You have 50 minutes to complete the quiz.

Problem 1

The system shown below is subjected to an initial excitation and allowed to vibrate freely.



$$\begin{aligned} \ell &= 0.3 \text{ m} \\ k &= 2000 \text{ N/m} \\ c &= 11.43 \text{ N} \cdot \text{s/m} \\ m &= \frac{0.3}{10} \text{ kg} \\ J_o &= \frac{m\ell^2}{3} = 0.3 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

- (10 points) Derive the equation of motion for the free vibration response of system [for $\theta(t)$].
- (5 points) Compute the frequency of the free vibration response.
- (10 points) If the initial angular displacement and velocity of the bar are $\theta(0) = 0.03 \text{ rad}$ and $\dot{\theta}(0) = \frac{2\pi}{0.257} \text{ rad/s}$, compute the amplitude and phase angle of the free vibration response; i.e. if the response is:

$$\theta(t) = \Theta e^{-\zeta \omega_n t} \cos(\omega_d t - \phi)$$

compute Θ and ϕ .

(A.)

$$-J_o \ddot{\theta} - (c \dot{\theta} l) l - [k \theta (0.7l)] 0.7l = 0$$

$$\boxed{J_o \ddot{\theta} + c l^2 \dot{\theta} + 0.49 k l^2 \theta = 0}$$

(B.)

$$\ddot{\theta} + \frac{c l^2}{J_o} \dot{\theta} + \frac{0.49 k l^2}{J_o} \theta = 0 \quad \omega_n^2 = \frac{0.49 k l^2}{J_o} \quad 2\zeta \omega_n = \frac{c l^2}{J_o}$$

$$\omega_n = 0.7l \sqrt{\frac{k}{J_o}} = 0.7(0.3 \text{ m}) \sqrt{\frac{2000 \text{ N/m}}{0.3 \text{ kg} \cdot \text{m}^2}} = 17.14 \text{ rad/s}$$

$$\zeta = \frac{c l^2}{2 J_o \omega_n} = \frac{11.43 \text{ N} \cdot \text{s/m} (0.3 \text{ m})^2}{2 (0.3 \text{ kg} \cdot \text{m}^2) (17.14 \text{ rad/s})} \therefore 0.10 \text{ underdamped}$$

$$\Rightarrow \omega_d = \omega_n \sqrt{1 - \zeta^2} = 17.14 \text{ rad/s} \sqrt{1 - (0.1)^2} \quad \boxed{17.05 \text{ rad/s}}$$

$$(c.) \quad \theta(t) = e^{-\zeta\omega_n t} [A_1 \cos\omega_d t + B_1 \sin\omega_d t]$$

$$\theta(0) = 0.03 \text{ rad} = A_1$$

$$\dot{\theta}(0) = 0.2\pi \text{ rad/s} = \left(\begin{array}{l} e^{-\zeta\omega_n t} [\omega_d A_1 \sin\omega_d t + \omega_d B_1 \cos\omega_d t] \\ - \zeta\omega_n e^{-\zeta\omega_n t} [A_1 \cos\omega_d t + B_1 \sin\omega_d t] \end{array} \right) \Bigg|_{t=0}$$

$$0.2\pi \text{ rad/s} = \omega_d B_1 - \zeta\omega_n A_1$$

$$B_1 = \frac{0.2\pi \text{ rad/s} - 0.1(17.14 \text{ rad/s})(0.03 \text{ rad})}{17.05 \text{ rad/s}}$$

$$B_1 = 0.034 \text{ rad}$$

$$\theta(t) = \Theta e^{-\zeta\omega_n t} \cos(\omega_d t - \phi)$$

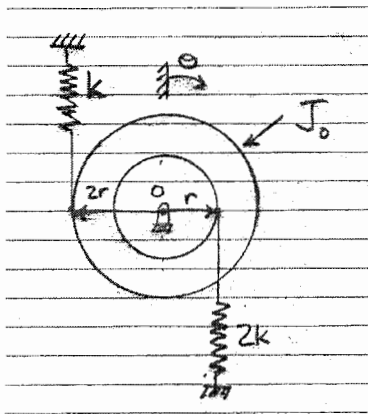
$$\Theta = \sqrt{A_1^2 + B_1^2} = \sqrt{(0.03)^2 + (0.034)^2} \text{ rad}$$

$$\boxed{\Theta = 0.045 \text{ rad}}$$

$$\phi = \tan^{-1}\left(\frac{B_1}{A_1}\right) = \tan^{-1}\left(\frac{0.034}{0.03}\right) = \boxed{\begin{array}{l} 0.85 \text{ rad} \\ (48.6 \text{ deg}) \end{array}}$$

Problem 2

For the system shown below:

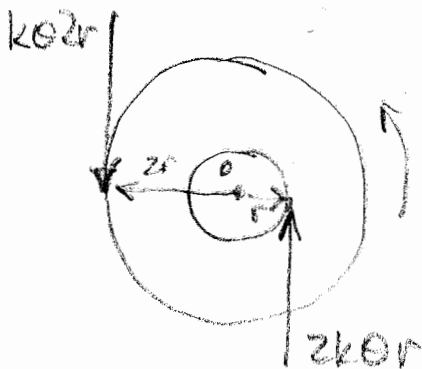


$$r = 0.05 \text{ m}$$

$$k = 5000 \text{ N/m}$$

$$J_o = 0.1 \text{ kg} \cdot \text{m}^2$$

- A. (10 points) Derive the equation of motion of the system.
 B. (5 points) Compute the undamped natural frequency.



(A.)
$$-J_o \ddot{\theta} - (2kr)\theta r - (k\theta r)(2r) = 0$$

$$\boxed{J_o \ddot{\theta} + 6kr^2 \theta = 0}$$

(B.)
$$\omega_n^2 = \frac{6kr^2}{J_o}$$

$$\omega_n = \sqrt{\frac{6kr^2}{J_o}} = 0.135 \text{ s}^{-1} \sqrt{\frac{6(5000 \text{ N/m})}{0.1 \text{ kg} \cdot \text{m}^2}}$$

$$\boxed{\omega_n = 27.4 \text{ rad/s}}$$

AE/ME 455 Optional Quiz Formula Sheet

Undamped Free Vibrations

$$\ddot{x} + \omega_n^2 x = 0$$

$$x(t) = C \cos(\omega_n t - \phi)$$

$$x(t) = C \sin(\omega_n t + \phi_o)$$

$$x(t) = A \cos \omega_n t + B \sin \omega_n t$$

$$C = \sqrt{A^2 + B^2}; \quad \phi = \tan^{-1}\left(\frac{B}{A}\right); \quad \phi_o = \tan^{-1}\left(\frac{A}{B}\right)$$

Damped Free Vibrations

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

1. $\zeta < 1$ Underdamped Case:

$$x(t) = X e^{-\zeta\omega_n t} \cos(\omega_d t - \phi)$$

$$x(t) = X e^{-\zeta\omega_n t} \sin(\omega_d t + \phi_o)$$

$$x(t) = e^{-\zeta\omega_n t} [A_1 \cos \omega_d t + B_1 \sin \omega_d t]$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$X = \sqrt{A_1^2 + B_1^2}; \quad \phi = \tan^{-1}\left(\frac{B_1}{A_1}\right); \quad \phi_o = \tan^{-1}\left(\frac{A_1}{B_1}\right)$$

2. $\zeta = 1$ Critically Damped Case:

$$x(t) = (C_1 + C_2 t) e^{-\omega_n t}$$

3. $\zeta > 1$ Overdamped Case:

$$x(t) = (C_1 e^{(\sqrt{\zeta^2 - 1})\omega_n t} + C_2 e^{-(\sqrt{\zeta^2 - 1})\omega_n t}) e^{-\zeta\omega_n t}$$

Other Formulae:

$$\text{period} = \tau = \frac{1}{f} = \frac{2\pi}{\omega}$$

f = frequency in Hz

ω = frequency in rad/s