Clarkson University
Department of Mechanical and Aeronautical Engineering
AE/ME 455 Mechanical Vibrations
Spring 2004 – OPTIONAL QUIZ
Friday, March 5, 2004

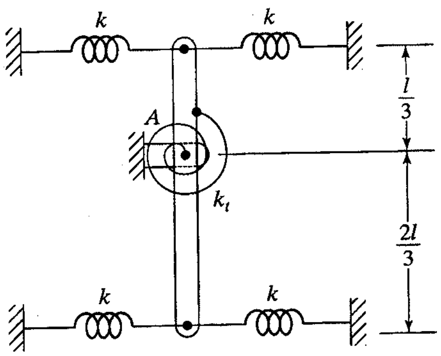
Name: SOLUTIONS

Student No.:______________________

Instructions: Read the problems carefully and space your time so as to gain the maximum number of points of credit. You will be given partial credit only for those steps in a solution that are leading to a logical conclusion. Please write neatly and clearly and make clear unambiguous sketches; cross out any work you do not wish to be considered. This quiz has 2 main problems with multiple parts worth 40 points total. The quiz is <u>CLOSED BOOK and CLOSED NOTES.</u> You have 50 minutes to complete the quiz.

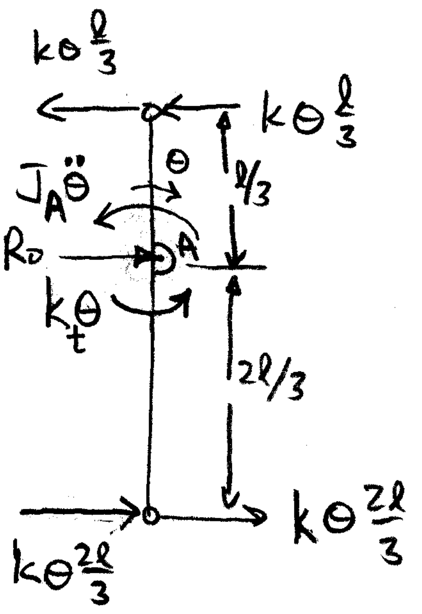
Problem 1

The system shown below has four linear springs, each with stiffness k, and a torsional spring at the axis of rotation A that has a torsional stiffness of k_t. Assume that the motion is unaffected by gravity (gravity is perpendicular to the plane of motion). The mass of the bar is m = 10 kg, the spring stiffness is k = 2000 N/m, the torsional spring stiffness is k_t = 1000 N·m/rad and the length is l = 5 m. The vibration of the system is excited by rotating the bar clockwise an amount θ = 0.09 rad and releasing it with an initial angular velocity of 0.5 rad/s.



A. (10 points) Compute the frequency at which the system vibrates.
B. (10 points) Compute the amplitude of the vibration.

Note: the mass moment of inertia of a slender bar of length l and mass m about an axis parallel to A passing through the center of mass is $J_{com}=ml^2/12$. The parallel axis theorum states that the mass moment of inertia for a parallel axis is $J_{axis}=J_{com}+mb^2$, where b is the distance between the axis passing through the center of mass and the parallel axis.



$$\Sigma M_A = -J_A\ddot{\theta} - 2k\theta\frac{\ell}{3}\cdot\frac{\ell}{3} - 2k\theta\frac{2\ell}{3}\cdot\frac{2\ell}{3} - k_t\theta = 0$$

$$\ddot{\theta} + \frac{1}{J_A}\left[\frac{10k\ell^2}{9} + k_t\right]\theta = 0$$

$$J_A = \frac{m\ell^2}{12} + m\left(\frac{\ell}{6}\right)^2 = \frac{m\ell^2}{9}$$

$$\ddot{\theta} + \left[10\frac{k}{m} + \frac{9k_t}{m\ell^2}\right]\theta = 0$$

PROB 1 CONT - $\ddot{\theta} + \omega_n^2 \theta = 0$

A. $\omega_n = \sqrt{10\frac{k}{m} + \frac{9k_t}{ml^2}}$

$$= \sqrt{10\left(\frac{2000\,N/m}{10\,kg}\right) + \frac{9(1000\,N{\cdot}m)}{(10\,kg)(5\,m)^2}}$$

$$\boxed{\omega_n = 45.12\ rad/s}$$

B. $\theta(t) = C_1 \cos\omega_n t + C_2 \sin\omega_n t$

$\theta(0) = 0.09\,rad = C_1$

$\dot{\theta}(0) = \omega_n C_2 = 0.5\,rad/s$

$$C_2 = \frac{0.5\,rad/s}{45.12\,rad/s} = 0.011\,rad$$

$\theta(t) = C\cos(\omega_n t - \phi)$

$$C = \sqrt{C_1^2 + C_2^2} = \sqrt{(.09)^2 + (.011)^2}\ rad$$

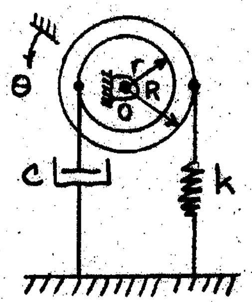
$$= \boxed{0.091\ rad}$$

Problem 2

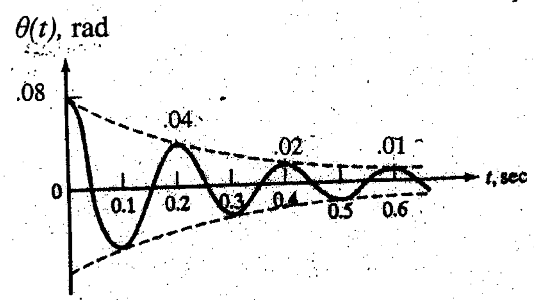
When excited by an initial excitation, the disk shown below vibrates with a frequency of 31.41 rad/s. The disk has a mass moment of inertia about its axis of rotation O of J_o = 0.0125 kg m^2. The response of the system to the initial excitation is shown.

A. (10 points) Compute the undamped natural frequency of the system.
B. (10 points) Compute the magnitude of the damping coefficient c.
C.

Hint – you are given enough information to do part A without determining the equation of motion. To do part B, you need the equation of motion.



$r = 3\text{cm} = .03\text{ m}$
$R = 5\text{cm} = .05\text{ m}$



A. UNDERDAMPED $\Rightarrow$ $\theta(t) = Xe^{-\zeta\omega_n t}\cos(\omega_d t - \phi)$

$\omega_d = 31.42$ rad/s GIVEN X:

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\frac{\theta(.2)}{\theta(.4)} = \frac{0.04}{0.02} = 2 = \frac{e^{-\zeta\omega_n\tau}}{e^{-\zeta\omega_n 2\tau}} = e^{-\zeta\omega_n\tau}$$

$$\tau = \frac{2\pi}{\omega_d} \Rightarrow 2 = e^{-\zeta 2\pi/\sqrt{1-\zeta^2}}$$

Prob 2 cont -

$$\delta = \ln(2) = -2\pi\zeta/\sqrt{1-\zeta^2}$$

$$\delta^2(1-\zeta^2) = 4\pi^2\zeta^2$$

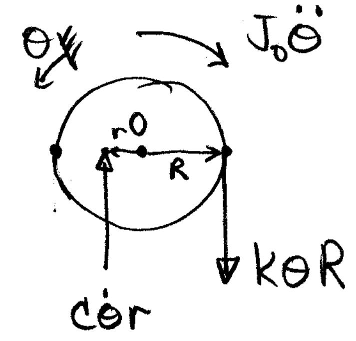
$$\delta^2 = (4\pi^2 + \delta)\zeta^2$$

$$\zeta = \sqrt{\frac{\delta^2}{4\pi^2+\delta}} = \frac{\ln(2)}{\sqrt{4\pi^2+\ln(2)}}$$

$$= 0.1094$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{31.41}{\sqrt{1-(0.1094)^2}} = \boxed{31.60\ \frac{rad}{s}}$$

B.



$$\Sigma M_o = -J_o\ddot{\theta} - c\dot{\theta}r^2 - k\theta R^2 = 0$$

$$\ddot{\theta} + \frac{cr^2}{J_o}\dot{\theta} + \frac{kR^2}{J_o}\theta = 0$$

$$2\zeta\omega_n = \frac{cr^2}{J_o}$$

$$\Rightarrow c = (2\zeta\omega_n)J_o/r^2 = \frac{2(.1094)(31.41\ rad/s)(.0125\ kg\cdot m^2)}{(0.03\ m)^2}$$

$N \cdot s/m = kg \cdot m/s^2 \; s/m = kg/s$

Prob 2 cont -

$$\Rightarrow \quad C = 95.45\ kg/s = \boxed{95.45\ N \cdot s/m}$$